Artificial Intelligence

Supervised Learning

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Learning

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specific observations $\rightarrow$ general rules

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Deduction
general axioms $\rightarrow$ specific propositions

(guaranteed to be correct)

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Example
the sun rose every morning in the past $\rightarrow$ the sun will rise tomorrow

Example
all squirrels are mortal and Scrat is a squirrel $\rightarrow$ Scrat is mortal

Forms of learning

Parameters

- *component* to be improved
- *prior knowledge* $\rightarrow$ *model*
- *data* and *feedback*

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Components

- A direct mapping from conditions on the current state to actions
- A means to infer relevant properties of the world from the percept sequence
- Information about the way the world evolves and about the results of possible actions
- Utility information indicating the desirability of world states
- ...

Forms of learning

Data
$(x_1, y_1), (x_2, y_2), \dots \in X \times Y$

- **Classification** : Y is *finite* (e.g. $\{sunny, cloudy, rainy\}$ or $\{true, false\}$)
- **Regression** : Y is *infinite* (e.g. $\mathbb{N}$)

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Feedback

- **Supervised learning** : the agent observes *input-output* pairs (x, y) and learn $y = f(x)$
- **Unsupervised learning** : the agent learns *pattern* from *inputs*
- **Reinforcement learning** : the agent learns from a serie of reinforcements : *rewards* and *punishments*

Content

Supervised Learning

Supervised Learning - framework

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Stationarity assumption

- $P(E_j) = P(E_{j+1}) = P(E_{j+2}) = \dots$: each example has the same prior probability distribution
- $P(E_j) = P(E_j | E_{j-1}, E_{j-2}, \dots)$: each example is independent from previous examples

$\hookrightarrow$ *independent and identically distributed*

Model and dataset

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Learn with part of the data and *evaluate* with the rest :

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k-fold cross-validation

- split the training set into k subsets
- iterate the three steps for all $i \in [1, k]$:
    - take subset i out
    - train with $k - 1$ joint subsets
    - validate with the subset i

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$y = f(x)$ and $\hat{y} = h(x)$

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example : For a spam filter, $L(spam, nospam) = 1$ and $L(nospam, spam) = 10$

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(with $|E| = N$)

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$$\boxed{Cost(h) = EmpLoss(h) + \lambda Complexity(h)}$$

$\hat{h}^* = \underset{h \in \mathcal{H}}{argmin} Cost(h)$

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Realizability/Intractability

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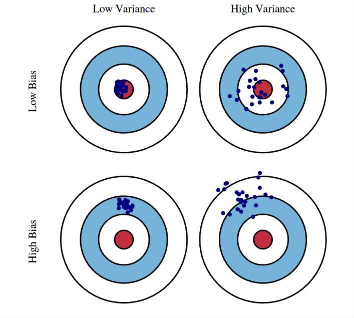
Bias-Variance tradeoff



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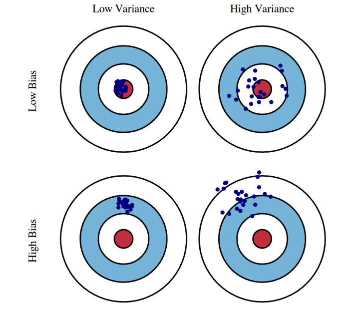

- complex low-bias hypotheses that fit the training data well
- simple low-variance hypotheses that generalize better

Model classes

- Decision trees

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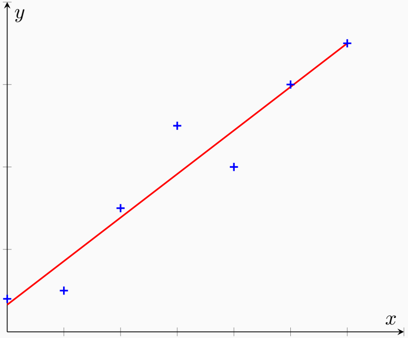
Linear Regression and Classification

Univariate linear regression



Sample set
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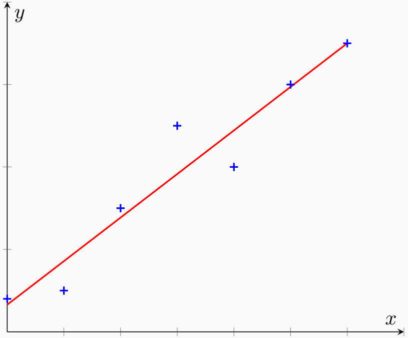
Univariate linear regression



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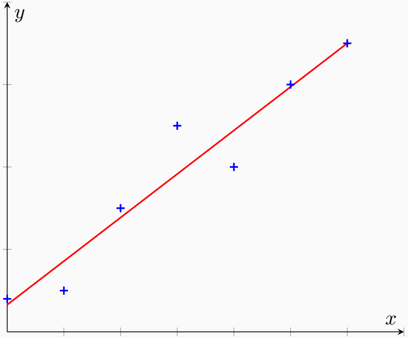

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Minimize loss

Normally distributed noise $\rightarrow L_2$ (Gauss)

$$Loss(h_{\vec{w}}) = \sum_{j=1}^{N} L_2(y_j, h_{\vec{w}}(x_j)) = \sum_{j=1}^{N} (y_j - (w_0 + w_1 x_j))^2$$

Minimize $L(\vec{w}) = Loss(h_{\vec{w}})$

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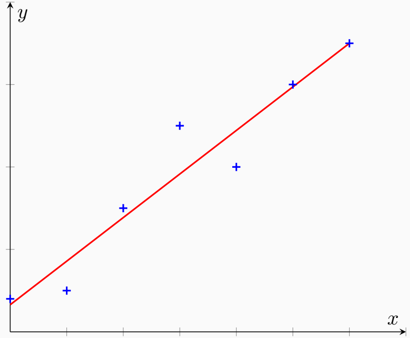

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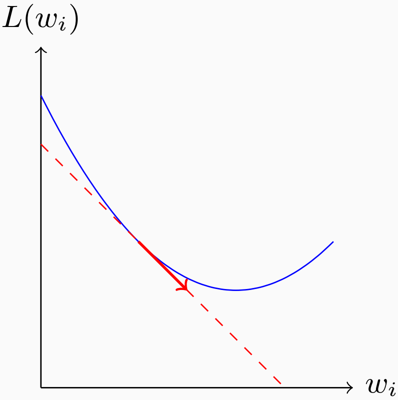
Analytic solution
Show that the minimum of $L(\vec{w})$ is obtained for :
$w_1 = \frac{(\sum x_j)(\sum y_j) - N(\sum x_j y_j)}{(\sum x_j)^2 - N(\sum x_j^2)}$ and $w_0 = \frac{(\sum y_j) - w_1(\sum x_j)}{N}$

Gradient descent



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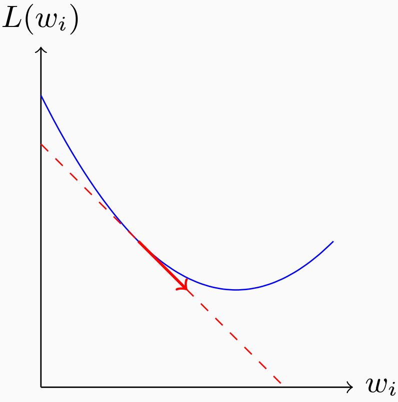

Algorithm

$$\boxed{w_i \leftarrow w_i - \alpha \frac{\partial L(\vec{w})}{\partial w_i}}$$ with α the *learning rate*

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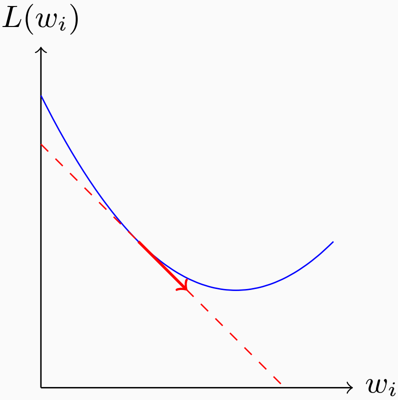

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$$w_1 \leftarrow w_1 + \alpha \sum_{j=1}^{N} (y_j - h_{\vec{w}}(x_j))x_j$$

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- $\vec{w} = (w_0, w_1, \ldots, w_d) \in \mathbb{R}^{d+1}$
- $\vec{x_j} = (x_{j1}, x_{j2}, \ldots, x_{jd}) \in \mathbb{R}^d$
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Analytic solution

$\boldsymbol{X}$: matrix of inputs (each row is an $\vec{x_j}$),

$\boldsymbol{y}$: vector of outputs (each row is a y_j)

$$L(\boldsymbol{w}) = \|\boldsymbol{X}.\boldsymbol{w} - \boldsymbol{y}\|^2$$

$$\nabla_{\boldsymbol{w}} L(\boldsymbol{w}) = 2\boldsymbol{X}^\top.(\boldsymbol{X}.\boldsymbol{w} - \boldsymbol{y}) = \mathbf{0}$$

$\boxed{\boldsymbol{w}^* = (\boldsymbol{X}^\top.\boldsymbol{X})^{-1}.\boldsymbol{X}^\top.\boldsymbol{y}}$: *normal equation*

Epoch poc poc

Batch gradient descent

$w_i \leftarrow w_i - \alpha \sum_{j=1}^{N} (y_j - h_{\vec{w}}(\vec{x_j}))x_{ji}$ (also called *deterministic gradient descent*)

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Stochastic gradient descent (SGD)

1. select and remove a *minibatch* of m out of N training examples (randomly)
2. compute a step $w_i \leftarrow w_i - \alpha \sum_{j=1}^{m} (y_j - h_{\vec{w}}(\vec{x_j}))x_{ji}$
3. iterate until no more training examples

Epoch poc poc

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2. compute a step $w_i \leftarrow w_i - \alpha \sum_{j=1}^{m} (y_j - h_{\vec{w}}(\vec{x_j}))x_{ji}$
3. iterate until no more training examples

Epoch
A step that covers all N training examples

Epoch poc poc

Batch gradient descent

$w_i \leftarrow w_i - \alpha \sum_{j=1}^{N} (y_j - h_{\vec{w}}(\vec{x_j}))x_{ji}$ (also called *deterministic gradient descent*)

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Complete algorithm
Iterate E epochs until *convergence*.

Regularization

Overfitting
In high-dimensional spaces, some irrelevant dimension might appear to be useful

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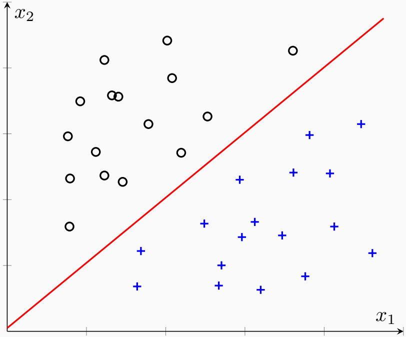
For linear functions

$$Complexity(h_{\vec{w}}) = L_q(\vec{w}) = \sum_{i=0}^{d} |w_i|^q$$

Usually, we use $q = 1$: L_1 regularization $\rightarrow$ produces *sparse model* (remove attributes)

Linear classification



Sample set
$\{(\vec{x_1}, y_1), (\vec{x_2}, y_2), \ldots, (\vec{x_N}, y_N)\} \subseteq \mathbb{R}^d \times \{0, 1\}$

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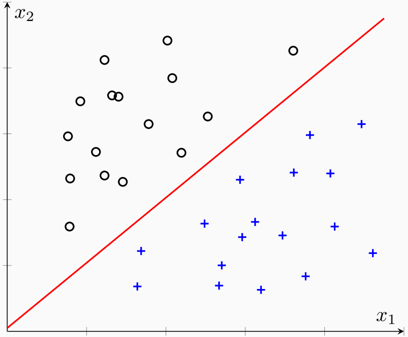

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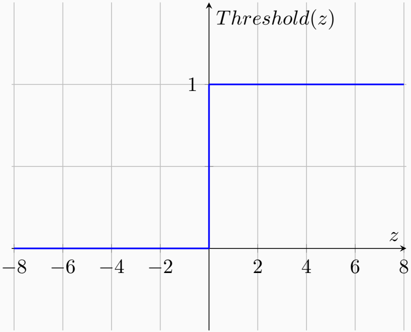
Hypothesis
The *decision boundary* is a linear separator.

Hard threshold linear classifier



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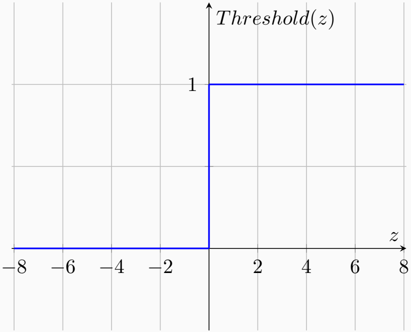

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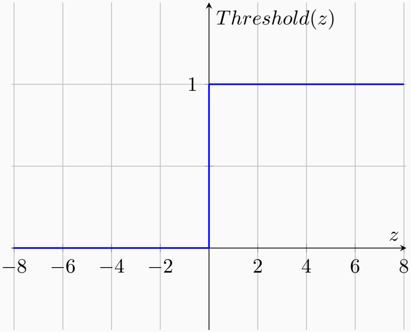

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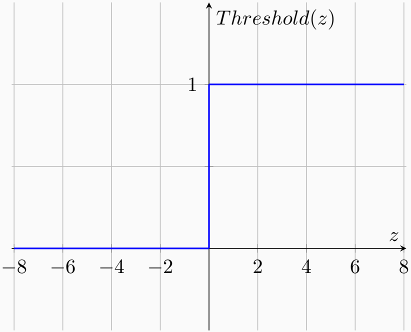

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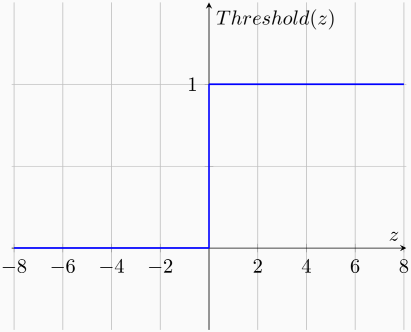

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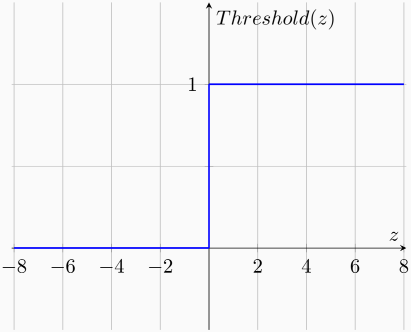

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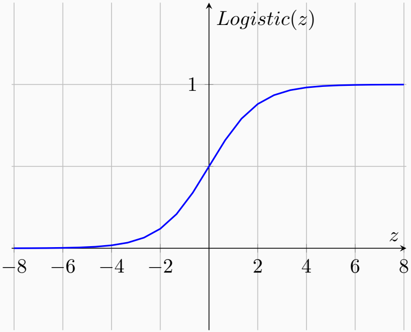
Technically, we require that :

$\sum_{t=1}^{\infty} \alpha(t) = \infty$ and $\sum_{t=1}^{\infty} \alpha(t)^2 < \infty$

Logistic linear classifier



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μ : location parameter (here $\mu = 0$)

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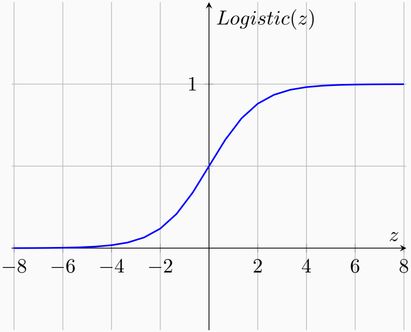

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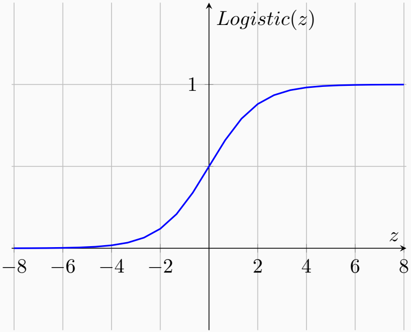

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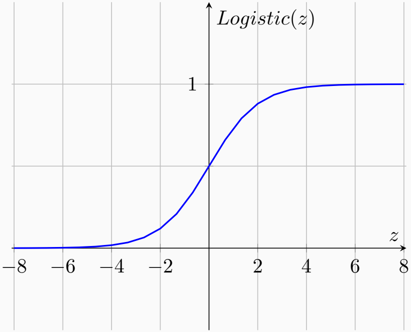

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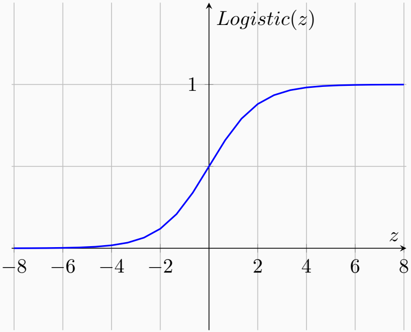

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The kernel trick

What if ..
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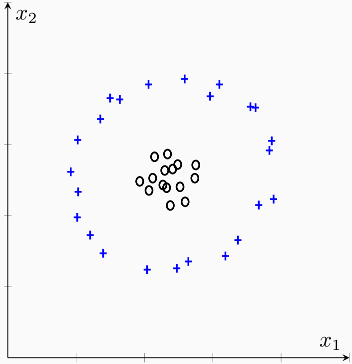

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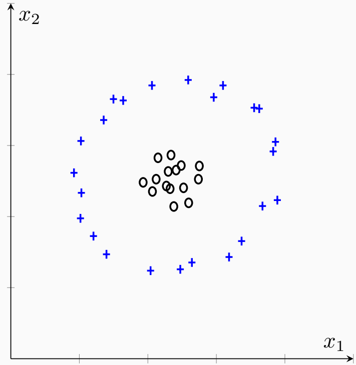

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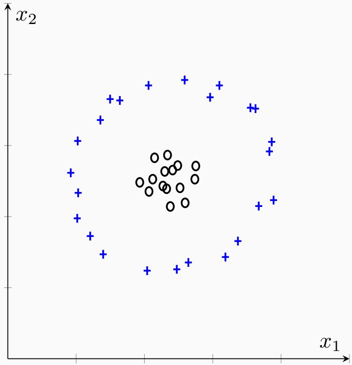

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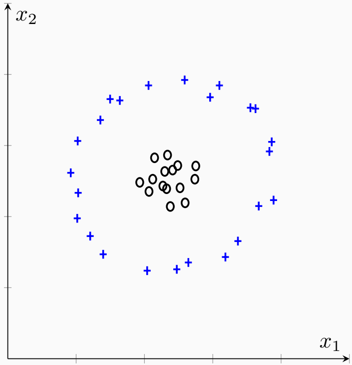

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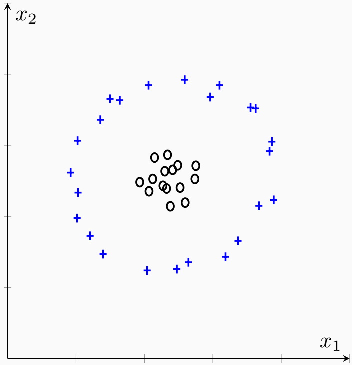

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Reformulation

We can show that $\vec{w} = \sum_{k=1}^{N} \delta_k \vec{x_k}$ then

$$h_{\vec{w}}(\vec{x_j}) = \sum_{k=1}^{N} \delta_k \vec{x_k}.\vec{x_j} \mapsto \sum_{k=1}^{N} \delta_k \phi(\vec{x_k}).\phi(\vec{x_j}) = \sum_{k=1}^{N} \delta_k K(\vec{x_k}, \vec{x_j})$$

Linear regression in the new space

$$h_{\vec{w}}(\phi(\vec{x_j})) = \sum_{k=1}^{N} \delta_k K(\vec{x_k}, \vec{x_j})$$

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$K \in \mathbb{R}^N \times \mathbb{R}^N$ s.t $K_{ij} = K(\vec{x_i}, \vec{x_j})$

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We compute $\vec{\delta}$ instead of $\vec{w}$:

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Popular kernel functions

- Linear : $K(\vec{x}, \vec{z}) = \vec{x}.\vec{z}$
- Polynomial : $K(\vec{x}, \vec{z}) = (1 + \vec{x}.\vec{z})^d$
- Radial Basis Function (RBF) : $K(\vec{x}, \vec{z}) = e^{\frac{-\|\vec{x}-\vec{z}\|^2}{\sigma^2}}$
- Laplacian Kernel : $K(\vec{x}, \vec{z}) = e^{\frac{-\|\vec{x}-\vec{z}\|}{\sigma}}$
- Sigmoïd Kernel : $K(\vec{x}, \vec{z}) = tanh(a\vec{x}.\vec{z} + b)$

demo

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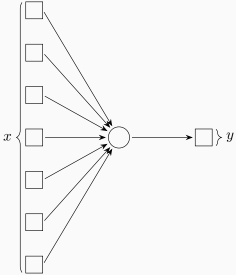
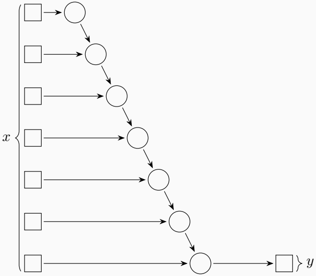
Deep Learning

Why is deep learning successful ?

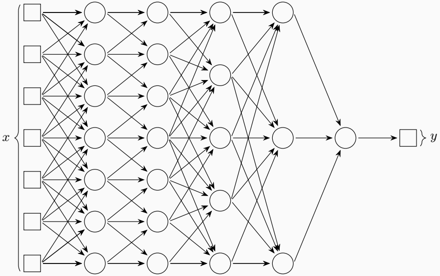
Linear regression



Decision list



Deep learning network



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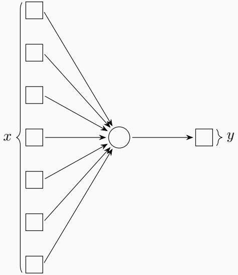

Shallow
Short computation path

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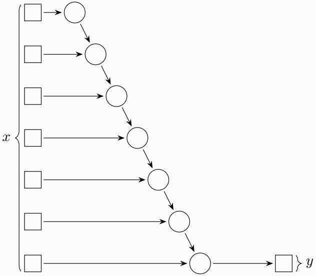
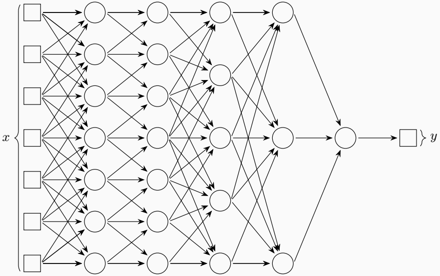


No interaction
No complex interaction between inputs

Deep learning network



Deep
Long computation path and complex interactions between many inputs

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Type of networks

- **feedforward network** : directed acyclic graph
- **recurrent network** : loops computing intermediate or final output

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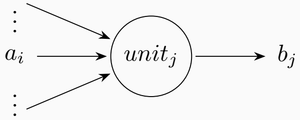

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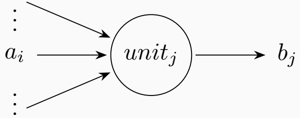

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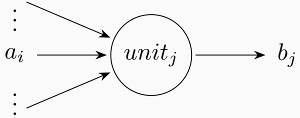

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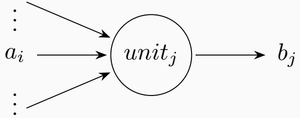

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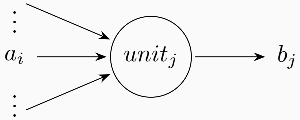

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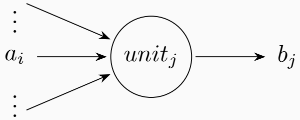

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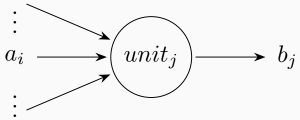
- **ReLU** (Rectified Linear Unit) : $ReLU(x) = \max(0, x)$
- **Softplus** (smooth ReLU) : $softplus(x) = log(1 + e^x)$
- **tanh** : $tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$ $(= 2\sigma(2x) - 1)$

Deep Learning

Type of networks

- **feedforward network** : directed acyclic graph
- **recurrent network** : loops computing intermediate or final output

Unit (a.k.a. Artificial Neuron)



$$\boxed{b_j = g_j(\sum_i w_{i,j} a_i)}$$

- g_j : **activation function** of unit j
- $\vec{w_j}$: weights of unit j

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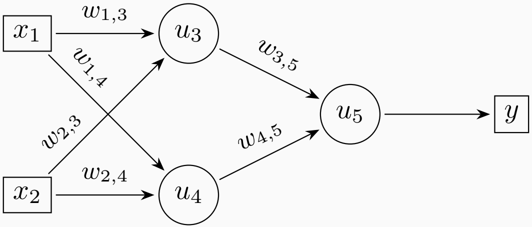
Universal approximation theorem

A network with just two layers (one non-linear and one linear) can approximate *any continuous function* to an *arbitrary degree of accuracy*.

An example

Example



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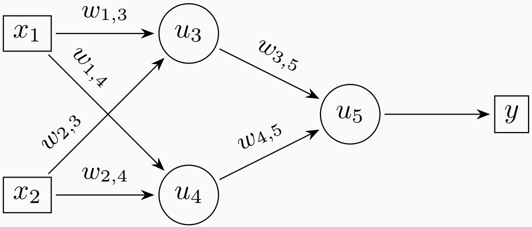

Forward computation

$$\hat{y} = g_5(w_{0,5} + w_{3,5}a_3 + w_{4,5}a_4)$$

$$\begin{aligned}\hat{y} = g_5(w_{0,5} &+ w_{3,5}g_3(w_{0,3} + w_{1,3}x_1 + w_{2,3}x_2) \\ &+ w_{4,5}g_4(w_{0,4} + w_{1,4}x_1 + w_{2,4}x_2))\end{aligned}$$

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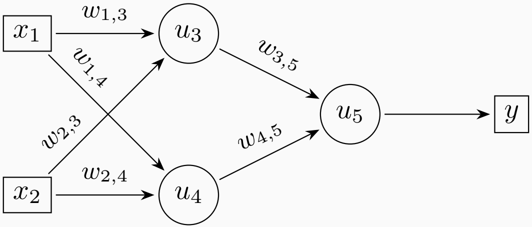

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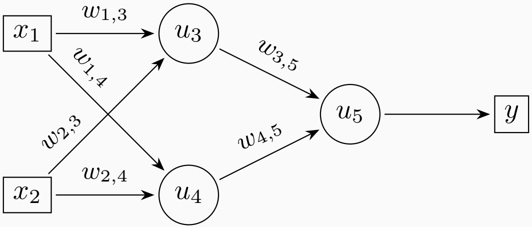

Gradient descent

$Loss(h_{\boldsymbol{W}}) = L_2(y, h_{\boldsymbol{W}}(\boldsymbol{x})) = (y - \hat{y})^2$

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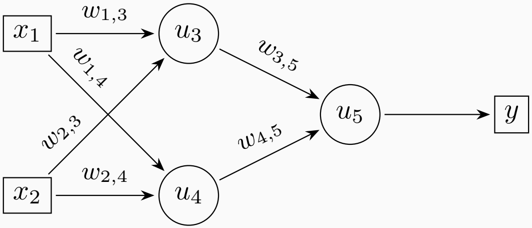

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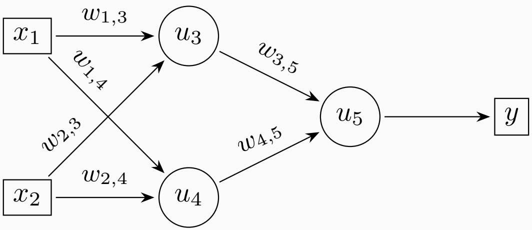

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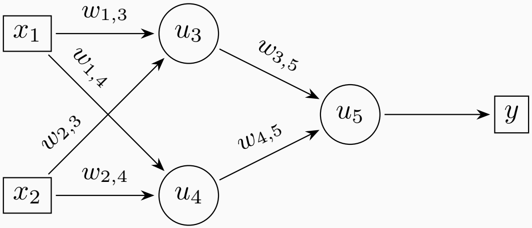

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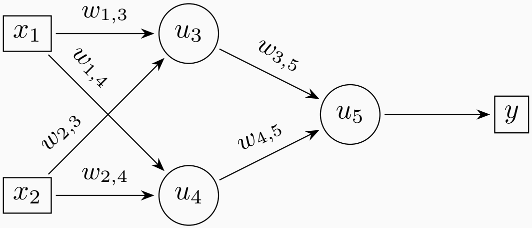

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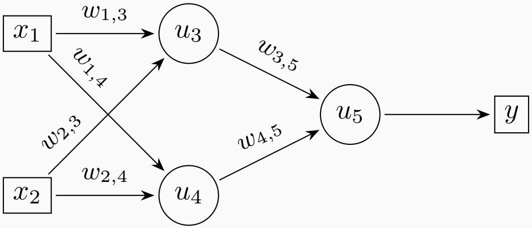

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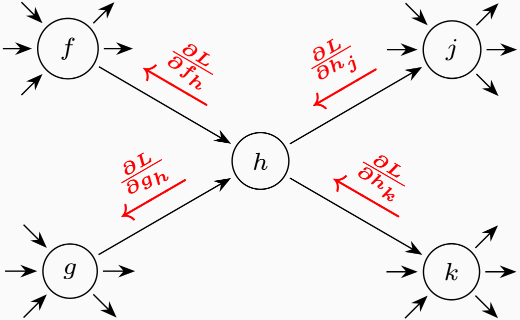
Vanishing gradient

When $g_i'(in_i) \approx 0 \rightarrow$ learning stops

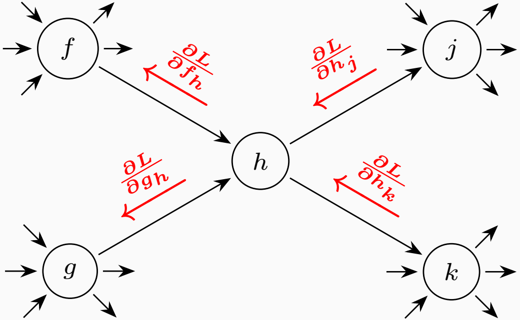
Backpropagation



h_j : message from node h to node j

$(h_j = h(f_h, g_h))$

Backpropagation



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Learning algorithms - Backpropagation

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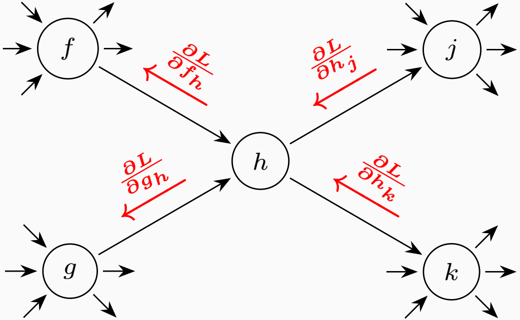

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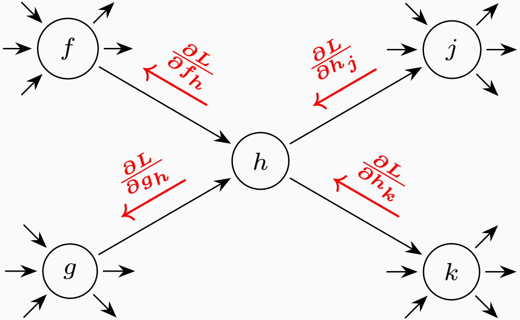

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Until ..

.. we reach a node corresponding to a parameter w : $\frac{\partial L}{\partial w} \rightarrow$ update w

Learning algorithms - Enhancements

General gradient descent
$\boldsymbol{W} \leftarrow \boldsymbol{W} - \alpha \nabla_{\boldsymbol{W}} L(\boldsymbol{W})$

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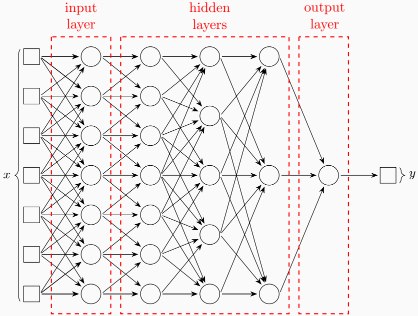
Batch normalization
For each example i of the minibatch, replace each output z_i of each node by $\hat{z}_i = \gamma \frac{z_i - \mu}{\sqrt{\varepsilon + \sigma^2}} + \beta$ (μ : mean, σ : standard deviation, within the minibatch) ($\varepsilon > 0$) (γ and β : new parameters)

Layers



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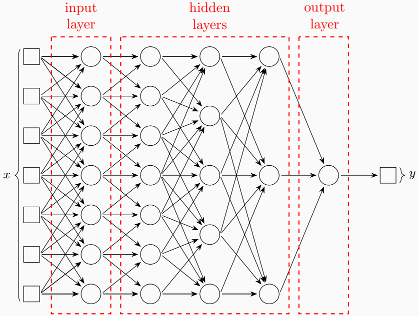

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- generally straighforward : $\{\top, \bot\} \rightarrow \{0, 1\}$, $\mathbb{R} \rightarrow \mathbb{R}$, *log* scale for big magnitudes, ...

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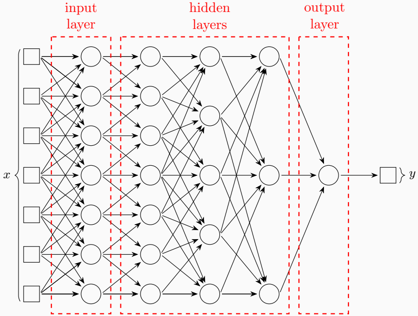

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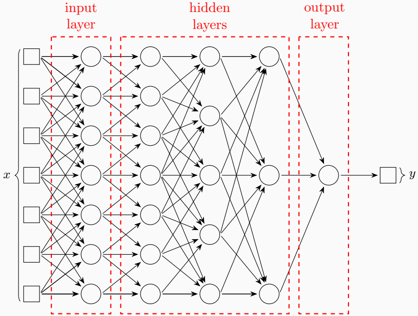

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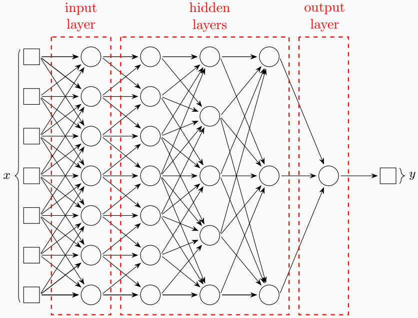

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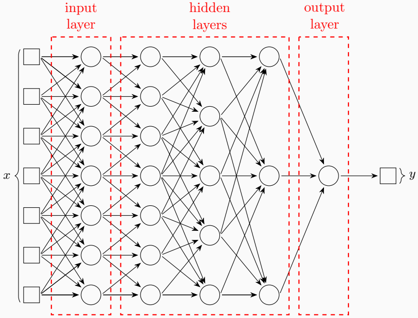

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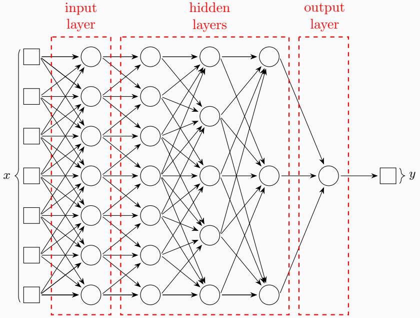

Hidden layer

- 1985-2010 : *sigmoid* or *tanh*
- now : *ReLU* and *softplus* more popular (vanishing gradient)

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Multiclass Classification
Interpret $\hat{y}$ as probabilities

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Interpret $\hat{y}$ as probabilities

Cross-entropy
Measure of dissimilarity between two distributions P and Q :

$$H(P, Q) = -E_{\boldsymbol{z} \sim P(\boldsymbol{z})}(\log Q(\boldsymbol{z})) = -\int P(\boldsymbol{z}) \log Q(\boldsymbol{z}) d\boldsymbol{z}$$

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Binary classification

- probability of output $y = 1$: $q_{y=1} = \hat{y}$
- probability of output $y = 0$: $q_{y=0} = 1 - \hat{y}$

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Cross-entropy loss

$$L(\boldsymbol{w}) = \frac{1}{N} \sum_{k=1}^{N} H(p_k, q_k)$$

$$\boxed{L(\boldsymbol{w}) = -\frac{1}{N} \sum_{k=1}^{N} (y_k \log \hat{y_k} + (1 - y_k) \log(1 - \hat{y_k}))}$$

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Convolutional Networks

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- **adjacency** $\rightarrow$ units should receive input from a *small local* region

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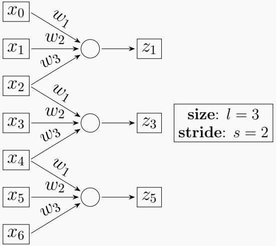
1D example



Convolution

- **kernel** : pattern of weights that is *replicated*

Convolutional Networks

Image specificities

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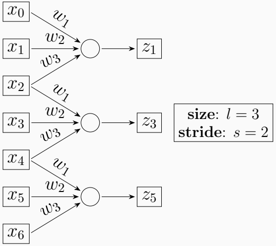
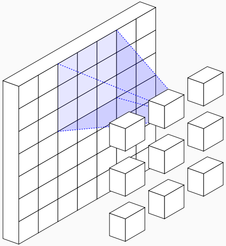


2D pattern



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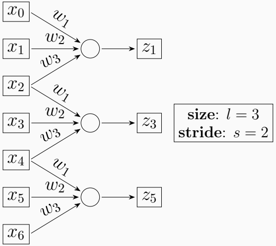

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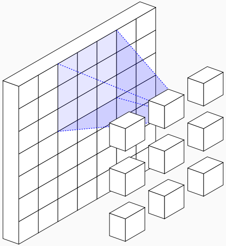

Pooling

- **average pooling** : $\boldsymbol{k} = (\frac{1}{l}, \ldots, \frac{1}{l})$ (if $s > 1$: *downsampling*)

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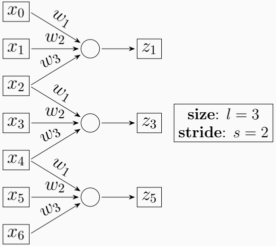

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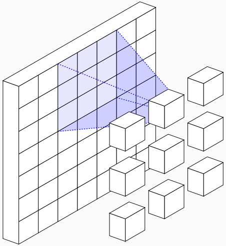

Pooling

- **average pooling** : $\boldsymbol{k} = (\frac{1}{l}, \ldots, \frac{1}{l})$ (if $s > 1$: *downsampling*)
- **max-pooling** : $z_i = \max_{1 \leq j \leq l}(x_{j+i-(l+1)/2})$

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Multidimensional arrays of any dimension :

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minibatch of 64 images RGB 256x256
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input	$\longrightarrow$	output
minibatch of 64 images RGB 256x256	96 kernels 5x5x3 with $s = 2$	**feature map**
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input	$\longrightarrow$	output
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256x256x3x64	$\longrightarrow$	128x128x96x64

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Idea
To avoid vanishing gradient in very deep networks $\rightarrow$ keep information of the previous layer

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Instead of $\boldsymbol{z}^{(i)} = h(\boldsymbol{z}^{(i-1)}) = g^{(i)}(\boldsymbol{W}^{(i)}\boldsymbol{z}^{(i-1)}) \rightarrow \boxed{\boldsymbol{z}^{(i)} = g_r^{(i)}(\boldsymbol{z}^{(i-1)} + f(\boldsymbol{z}^{(i-1)}))}$

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Disable a layer
We can make layers that can be disabled by setting $\boldsymbol{V} = \boldsymbol{0}$: if $g_r = ReLU$ (at least for layers $i-1$ and i), $\boldsymbol{z}^{(i-1)} = ReLU(\boldsymbol{in}^{(i-1)})$ then
$\boldsymbol{z}^{(i)} = ReLU(\boldsymbol{z}^{(i-1)}) = ReLU(ReLU(\boldsymbol{in}^{(i-1)})) = ReLU(\boldsymbol{in}^{(i-1)}) = \boldsymbol{z}^{(i-1)}$

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Time series
A sequence of inputs $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T$ and observed outputs $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T$.

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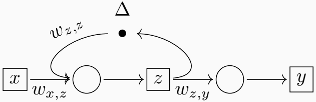

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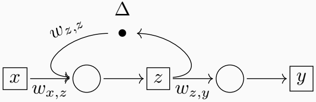

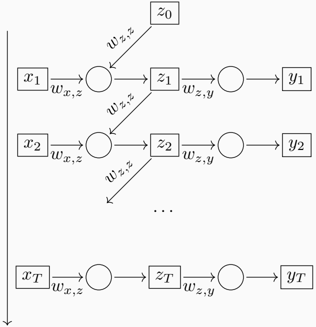

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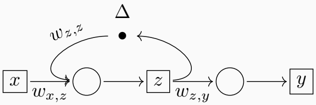

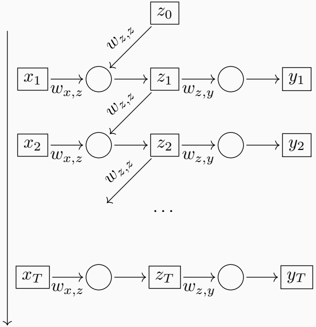

Forward

$z_t = g_z(w_{z,z} z_{t-1} + w_{x,z} x_t)$

and $\hat{y}_t = g_y(w_{y,z} z_t)$

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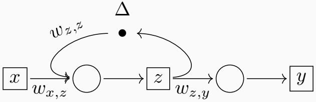

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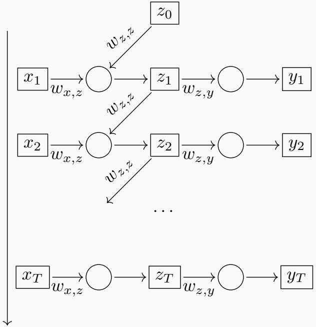

Backpropagation

$$\frac{\partial L}{\partial w_{z,z}} = \sum_{t=1}^{T} -2(y_t - \hat{y}_t) g_y'(in_{y,t}) w_{z,y} \frac{\partial z_t}{w_{z,z}}$$

$$\frac{\partial z_t}{\partial w_{z,z}} = g_z'(in_{z,t})(z_{t-1} + w_{z,z} \frac{\partial z_{t-1}}{w_{z,z}})$$

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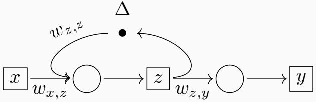

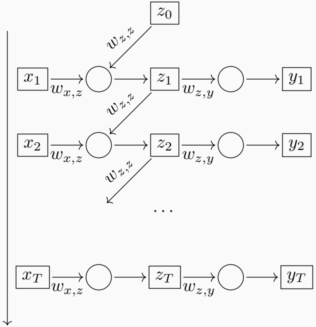

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Issue
Gradient at step T will include terms proportional to $w_{z,z} \prod_{t=1}^{T} g_z'(in_{z,t})$

$\hookrightarrow$ vanishing ($w_{z,z} < 1$) or *exploding* ($w_{z,z} > 1$) gradient

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- **memory cell** $\boldsymbol{c}$: *copied* at each time step

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- $\boldsymbol{z}_t = tanh(\boldsymbol{c}_t) \odot \boldsymbol{o}_t$

Improve generalization – Design the architecture

Specialized architecture

- **Convolutional** : images
- **Recurrent** : text and audio signals

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For a fixed number of weights : *the deeper the better*

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Train and evaluate
Reduce time of estimation : train on *test set* + evaluate on *validation set*

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- Smaller training set

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- Focus on subgraph

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Specialized architecture

- **Convolutional** : images
- **Recurrent** : text and audio signals

Neural architecture search
Optimisation problem with **hyperparameters** : depth, width, connectivity, . . .

- Grid search
- Evolutionary algorithm : recombination (joining parts of two networks) + mutation (adding/removing a layer or changing a parameter value)
- Hill climbing with mutations
- Reinforcement learning
- Bayesian optimization
- Gradient descent

Empirical result
For a fixed number of weights : *the deeper the better*

Train and evaluate
Reduce time of estimation : train on *test set* + evaluate on *validation set*

- Smaller training set
- Fewer batches + prediction of improvement
- Reduced version of the network
- Focus on subgraph
- Learn heuristic evaluation function

Improve generalization

Weight decay

Regularization with penalty $\boxed{\lambda \sum_{i,j} \boldsymbol{W}_{i,j}^2}$, typically $\lambda = 10^{-4}$

$\hookrightarrow$ Encourage small weights
(to stay in the linear part for sigmoid activation)

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Dropout
At each step of training *deactivate a random set of units*

- Encourage the detection of more features
- Make it more robust to noise

Neural Network Applications

Vision
Deep convolutional networks (since 1990s)

ImageNet competition : classification 1200000 images in 1000 categories

In 2012, *AlexNet* : error rate $< 15.3\%$ (2^{nd} : 25%) (now, error rate $< 2\%$)

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Translation problems :

- Two networks : from L1 to IR + from IR to L2
- One end-to-end network $\leftarrow$ performs better

Speech recognition : representation of words with high-dimensional vectors $\rightarrow$ *word embeddings*

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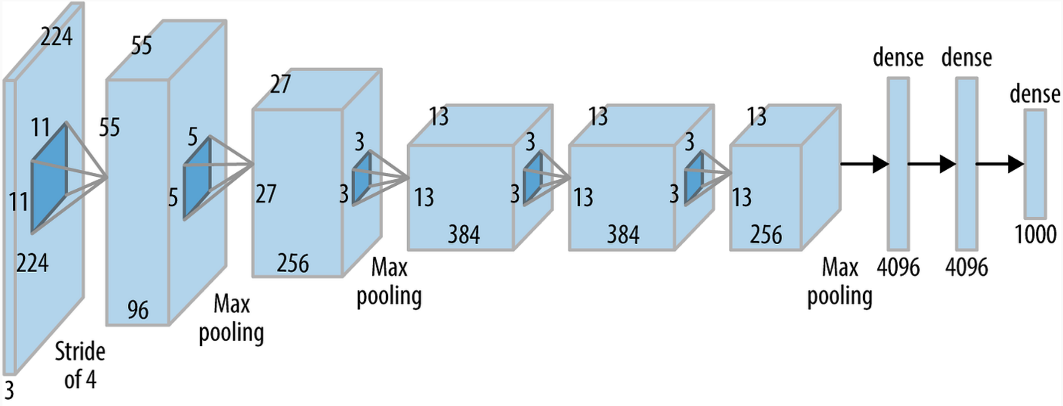
Reinforcement learning
Optimise the sum of *future rewards* : learn a value function, Q-function, policy, ...$\rightarrow$ *deep reinforcement learning*

DeepMind : *DQN* an Atari-playing agent (2013) and *AlphaGo* (2014)



Architecture of Alexnet. From left to right (input to output) five convolutional layers with Max Pooling after layers 1,2, and 5, followed by a three layer fully connected classifier (layers 6-8). The number of neurons in the output layer is equal to the designed number of output classes.

Deep Learning - Summary

- **Neural Networks** $=$ computation graph composed of parameterized linear-threshold units

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To go further ...

- **Transfer learning** : re-train a pretrained network for a specific task

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- **Generative Adversarial Networks** : a *generator* network + a *discriminator* network

Model Selection and Optimisation

Ensemble learning

Learn several hypothesis $h_1, h_2, \ldots, h_K$ and use a combination $h^* = \{h_1, h_2, \ldots, h_K\}$

- reduce **bias** of each base model by **combining**
- reduce **variance** of learning by **voting**

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$\boxed{h^*(\boldsymbol{x}) = \frac{1}{K}\sum_{i=1}^{K} h_i(\boldsymbol{x})}$: voting in the same model class

Example : **random forests**

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Boosting

1. Boost incorrectly classified training example by increasing its weight (number of occurences), iterate after learning each h_i
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Gradient boosting
Boosting with *gradient descent* to find the weight on training examples

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- **Logistic Regression** performs similarly than **SVM**
- **SVM** : is better for not too large dataset with high dimension
- **Deep Neural Network** : for complex pattern recognition (e.g. image or speech processing)

Data enhancement

- Not enough data $\rightarrow$ **data augmentation** (example : image cropping/rotating/...)

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- Unbalanced classes in data (example : unbalanced representation of negative vs. positive examples) $\rightarrow$ **undersample** or **oversample**
- **Outliers** : points far from the majority $\rightarrow$ some model classes are less susceptible : decision trees

Summary

Summary

- **Supervised learning** is learning on labelled datasets
- **Regression** is learning a function with infinite output values
- **Classification** is learning a function with finite output values
- **Linear/Logistic regression** is a simple yet powerful model class for supervised learning
- **Deep Neural Networks** are computation graphs composed of units made of a non-linear and a linear function
- **Deep learning** is well suited for visual object recognition, speech recognition, natural language processing and reinforcement learning

Sources

- Artificial Intelligence : A Modern Approach, *Stuart Russell* and *Peter Norvig*
- Lecture of *Didier Lime* (2022-2023)
- Lecture of *Kilian Weinberger* : `https://courses.cis.cornell.edu/cs4780/2017sp/`