Decidability problems for weak Time Petri Nets with read, reset and transfer arcs

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Outline

Weak and strong semantics for T-TPN

Weakly-strong TPN

Weak TPN with read, reset or transfer arcs

Conclusion

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Petri nets with time

Petri nets have been extended with time

In different ways

- date (point)[a] : Timed Petri nets
- interval[b] : Time Petri nets

[a] C. Ramchandani. "Analysis of asynchronous concurrent systems by timed Petri nets". PhD thesis. Cambridge, MA: Massachusetts Institute of Technology, 1974.

[b] P. M. Merlin. "A study of the recoverability of computing systems". PhD thesis. University of California, Irvine: Dep. of Information and Computer Science, 1974.

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Timing constraints are associated with

- places : P-timed or P-time
- arcs : A-timed or A-time
- transitions : T-timed or T-time
- tokens

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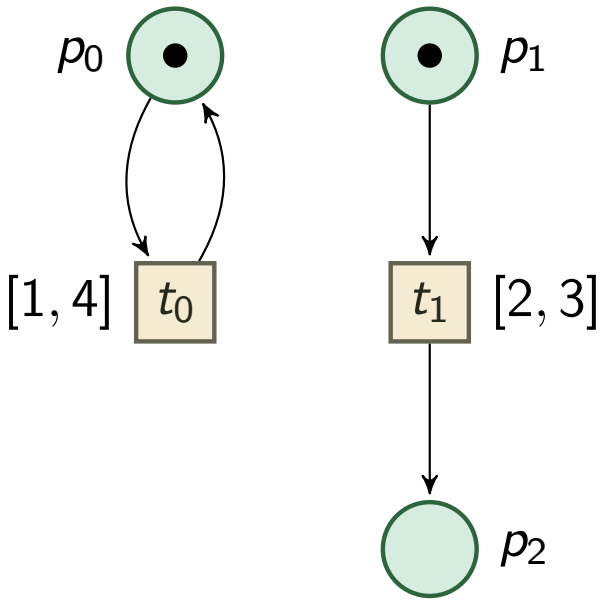
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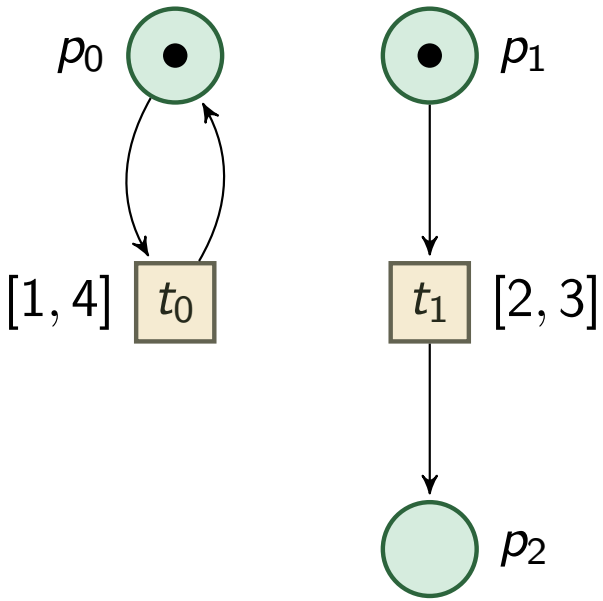
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T-Time Petri Nets (TPN)

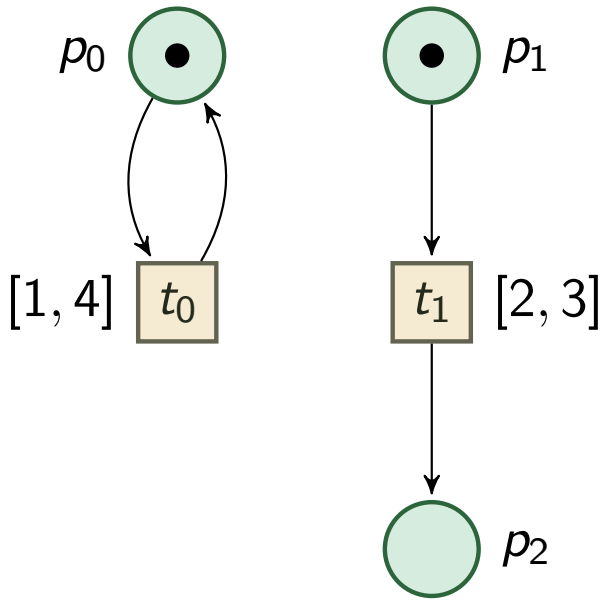


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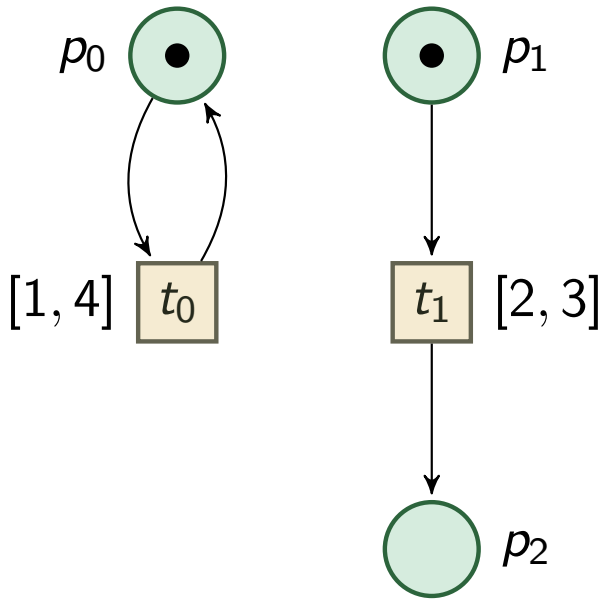
$t_0 \in [1, 4]$
$t_1 \in [2, 3]$

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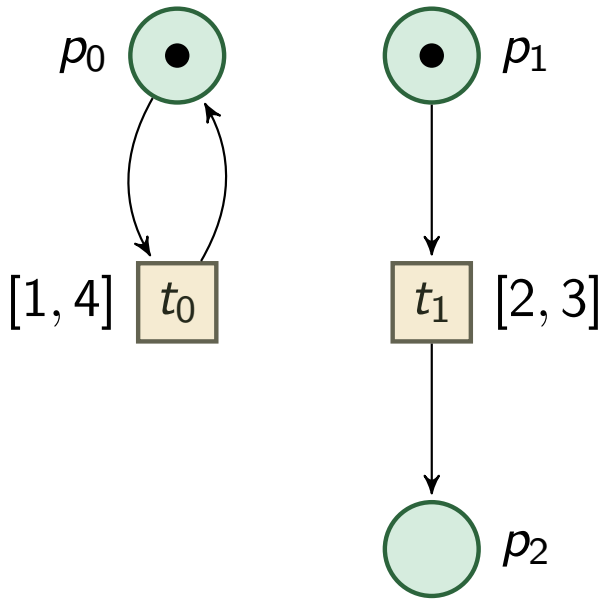
$$\begin{matrix} t_0 \in [1,4] \\ t_1 \in [2,3] \end{matrix} \xrightarrow{1.1} \begin{matrix} [0, 2.9] \\ [0.9,\ 1.9] \end{matrix}$$

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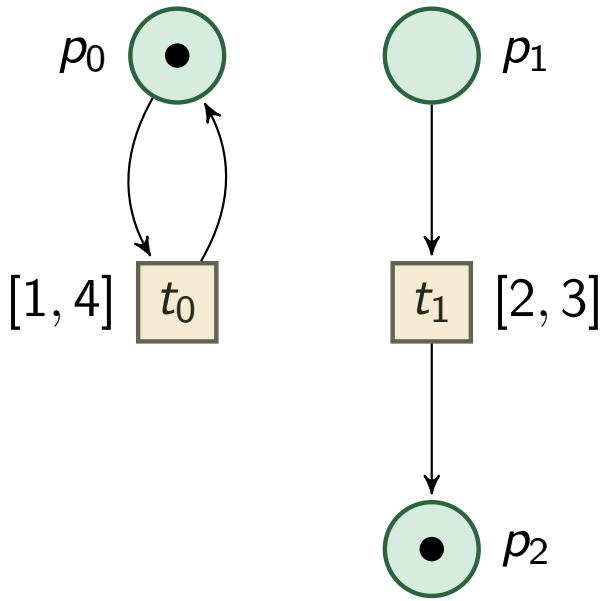
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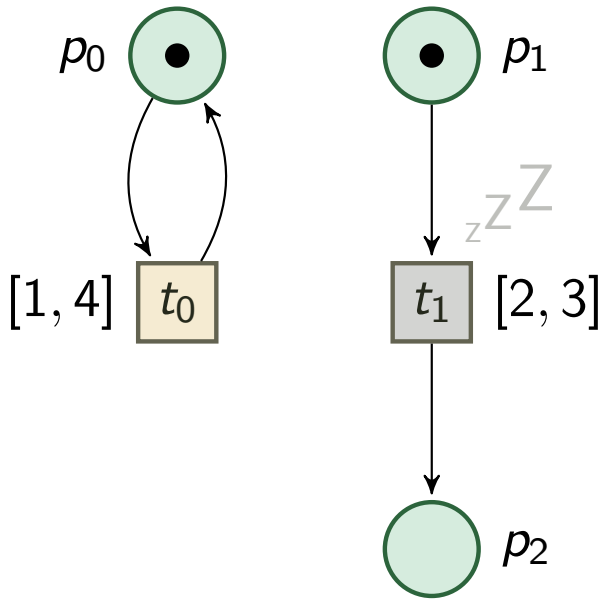
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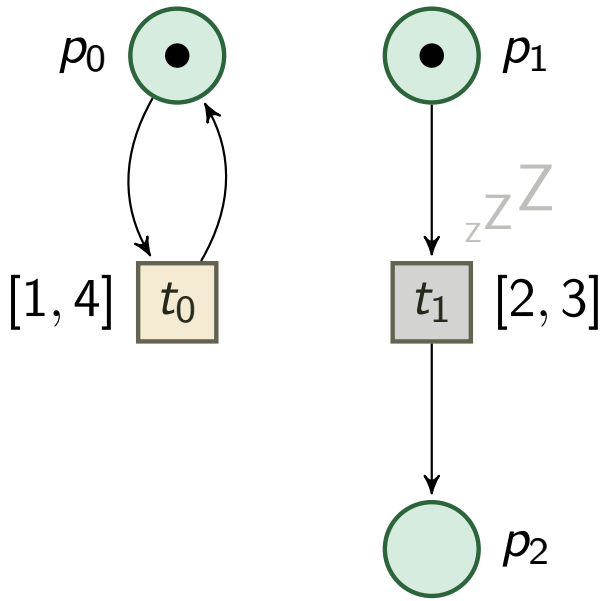


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$$\cdots \begin{matrix} [0, 2.1] \\ [0,\ 0] \end{matrix} \xrightarrow{1.1} \begin{matrix} [0,\ 1] \\ \text{sleep} \end{matrix}$$

t_1 falls asleep

T-Time Petri Nets (TPN)



strong semantics

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weak semantics

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Decidability problems and expressiveness

Decidability problems

- **Marking reachability**: Given m, $m \overset{?}{\in} \text{reach}(N)$
- **Marking coverability**: Given m, $\overset{?}{\exists}\, m'$ s.t. $m \leq m'$ and $m' \in \text{reach}(N)$
- **Boundedness**: $\overset{?}{\exists}\, m'$ s.t. $\forall m \in \text{reach}(N),\ m \leq m'$?

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Weak vs Strong semantics

- The three problems are **undecidable** for the strong semantics[a]
- The three problems are **decidable** for the weak semantics[b]

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Expressiveness

Weak and strong semantics are **incomparable**[a] (w.r.t. weak timed bisimilarity)

[a] M. Boyer and O. H. Roux. "On the compared expressiveness of arc, place and transition time Petri Nets". In: *Fundamenta Informaticae* (2008).

Outline

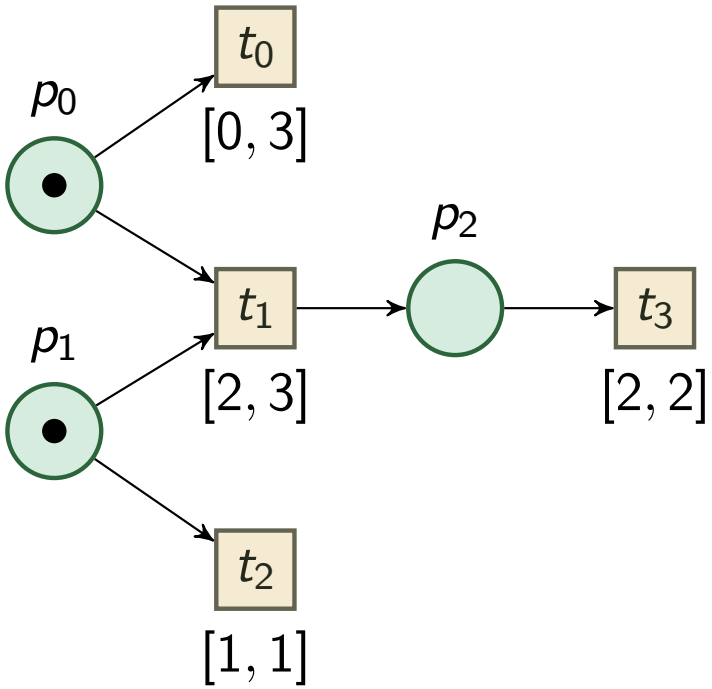
Weak and strong semantics for T-TPN

Weakly-strong TPN

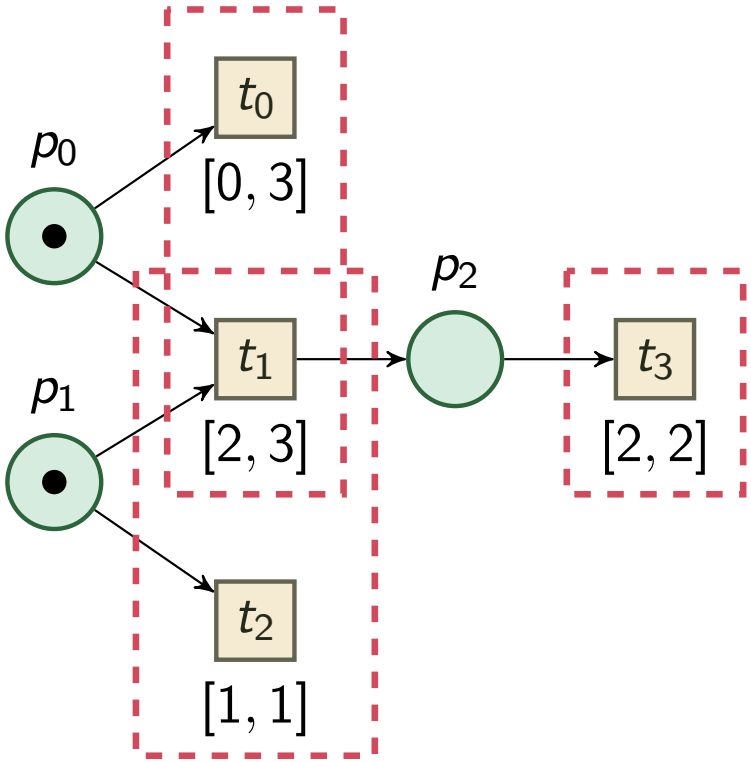
Weak TPN with read, reset or transfer arcs

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Weakly-strong TPN (WS-TPN)



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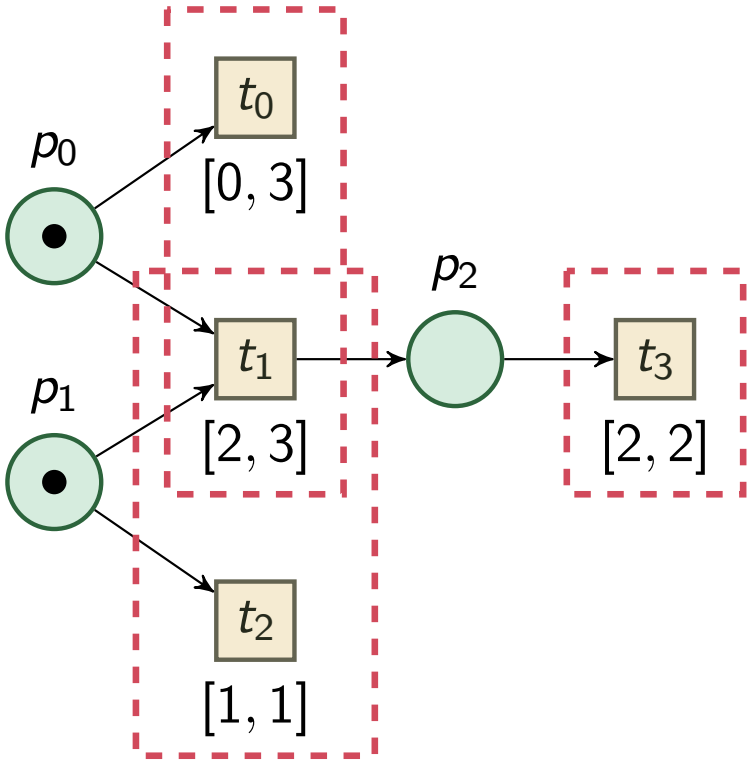


Packs

A pack = set of transitions

Here three packs: $\{t_0, t_1\}$, $\{t_1, t_2\}$ and $\{t_3\}$

Weakly-strong TPN (WS-TPN)



Packs

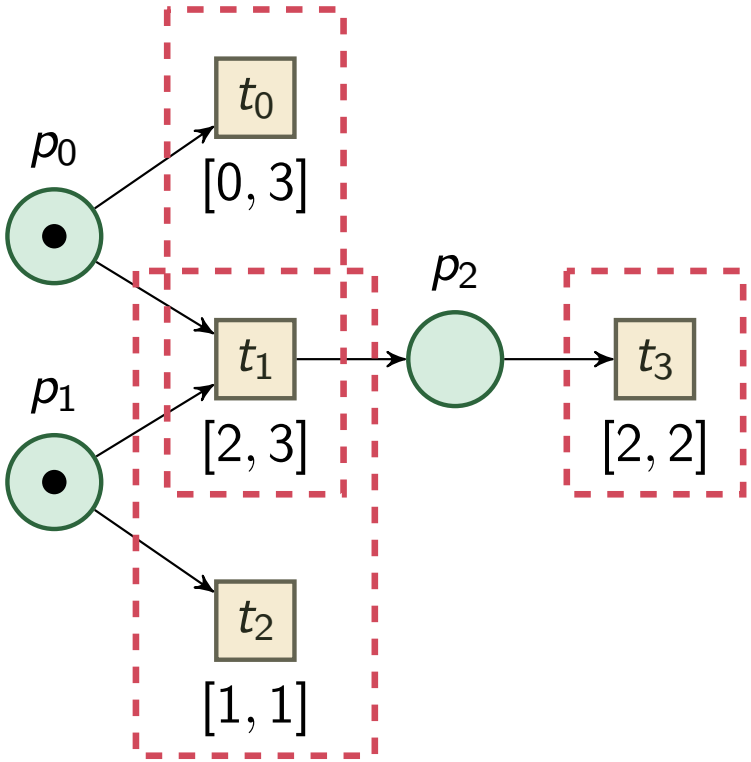
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Elapsing time condition

An enabled pack **cannot** fall asleep in its **enterity**

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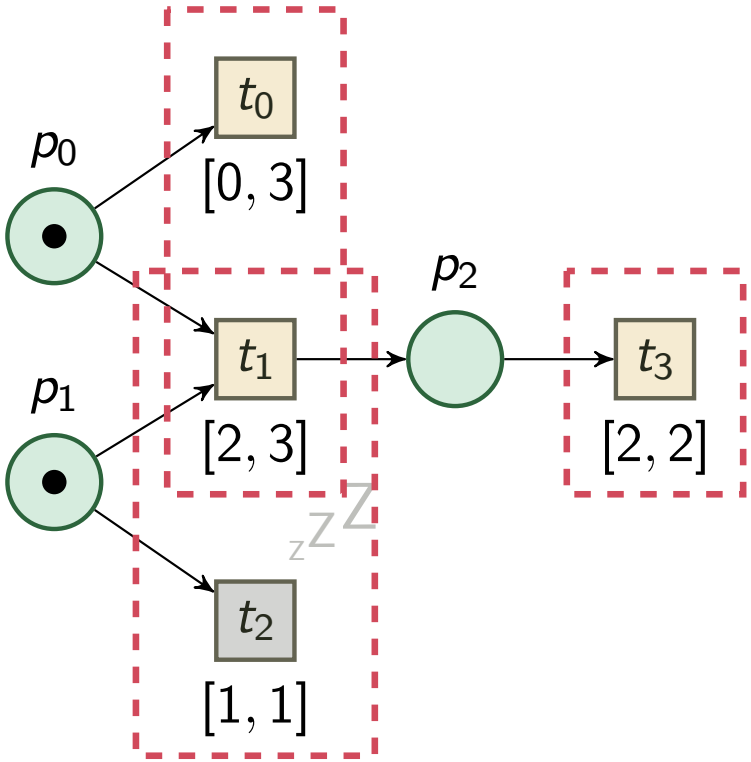
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$$t_0 \in [0, 3]$$
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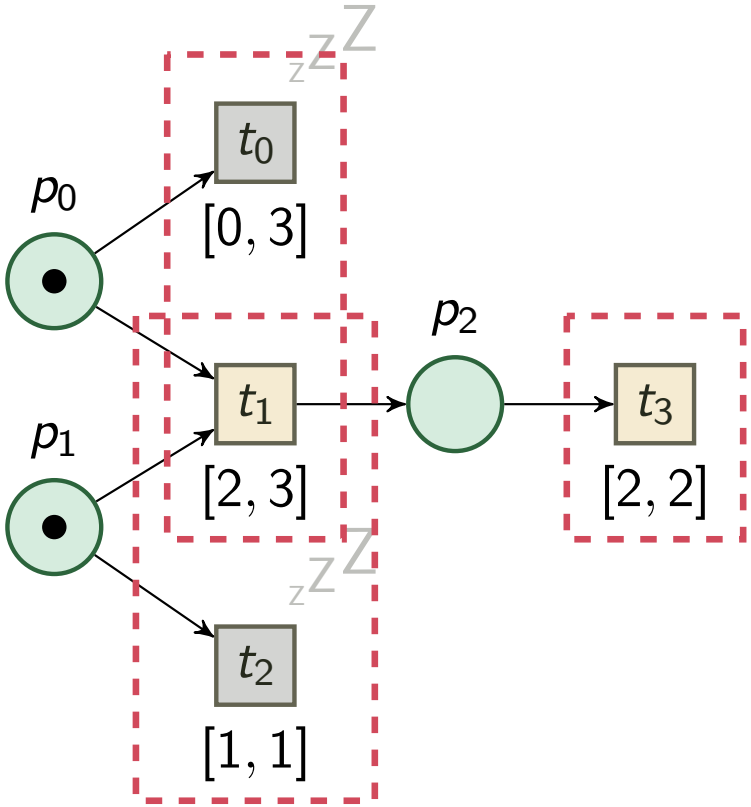
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$$\begin{array}{ll} t_0 \in [0,3] & [0, 1.9] \\ t_1 \in [2,3] \xrightarrow{1.1} & [1.9,\ 2.9] \\ t_2 \in [1,1] & \text{sleep} \\ t_3 \in - & - \end{array}$$

Weakly-strong TPN (WS-TPN)



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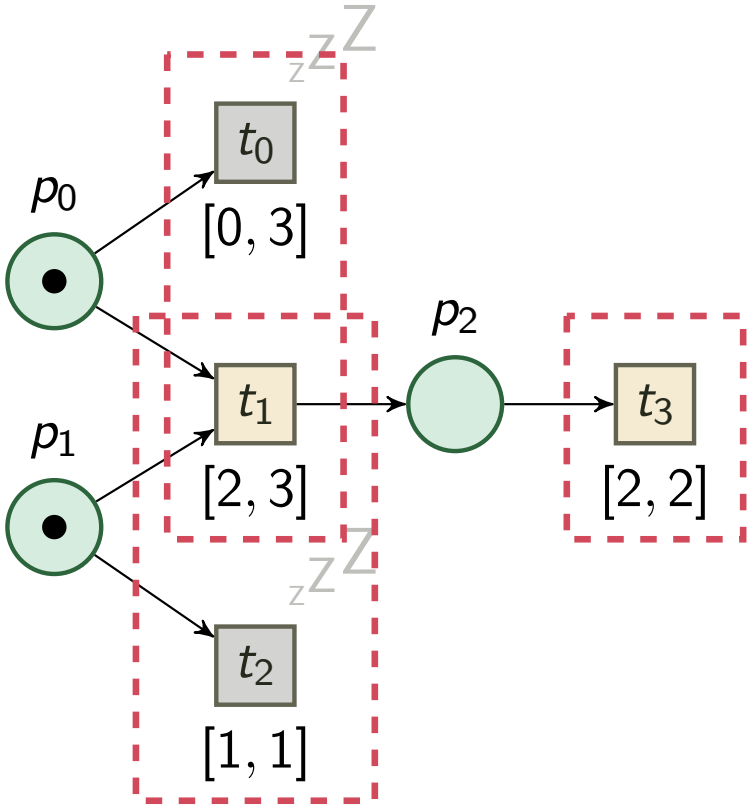
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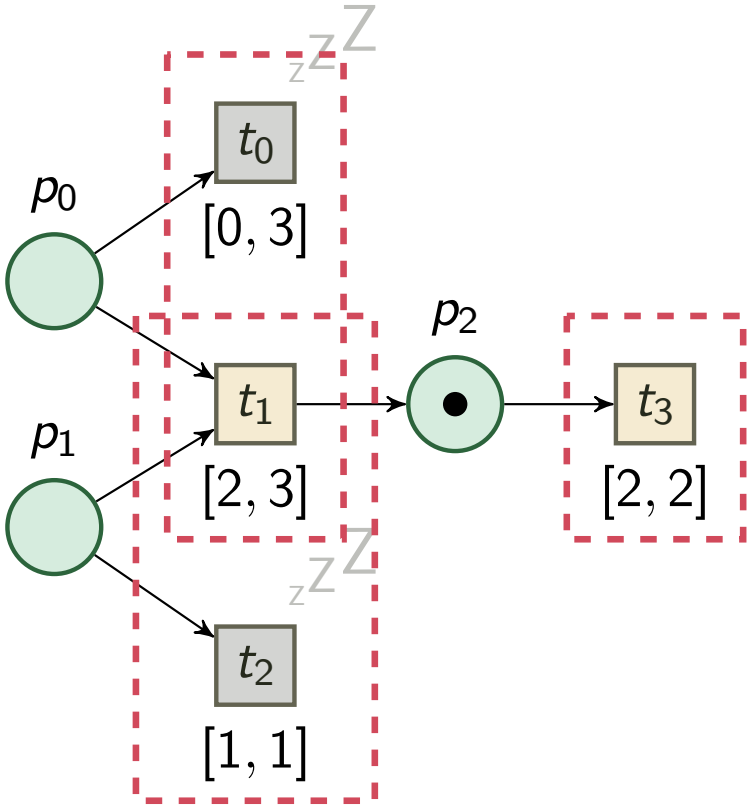
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Expressiveness of WS-TPN

Weak and strong semantics can be expressed

- **weak**: no pack
- **strong**: exactly one pack per transition

The limit for decidability

2-counter machine

- 2 counters: c_1 and c_2
- $\mathrm{inc}(l_j, c_i, l_k)$: increment
- $\mathrm{jzdec}(l_j, c_i, l_k, l_l)$: conditional decrement
- init: an initial line
- halt: a halting line

The halting problem

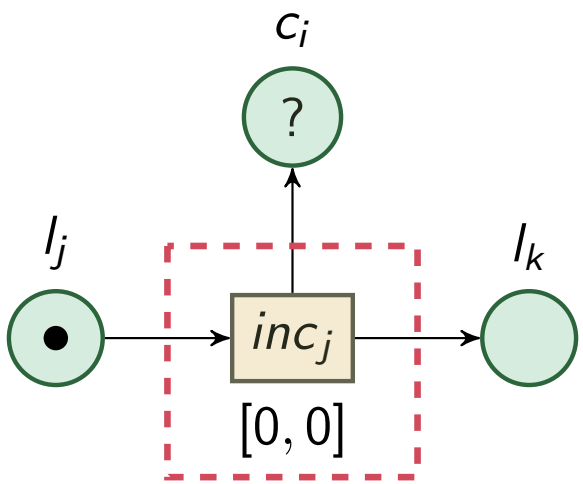
Given a 2-counter machine, decide if it can go from $(0, 0, \mathrm{init})$ to $(v_1, v_2, \mathrm{halt})$?
$\rightarrow$ **undecidable**[a]

[a] Marvin L. Minsky. "Computation: finite and infinite machines". In: (1967).

Standard technique

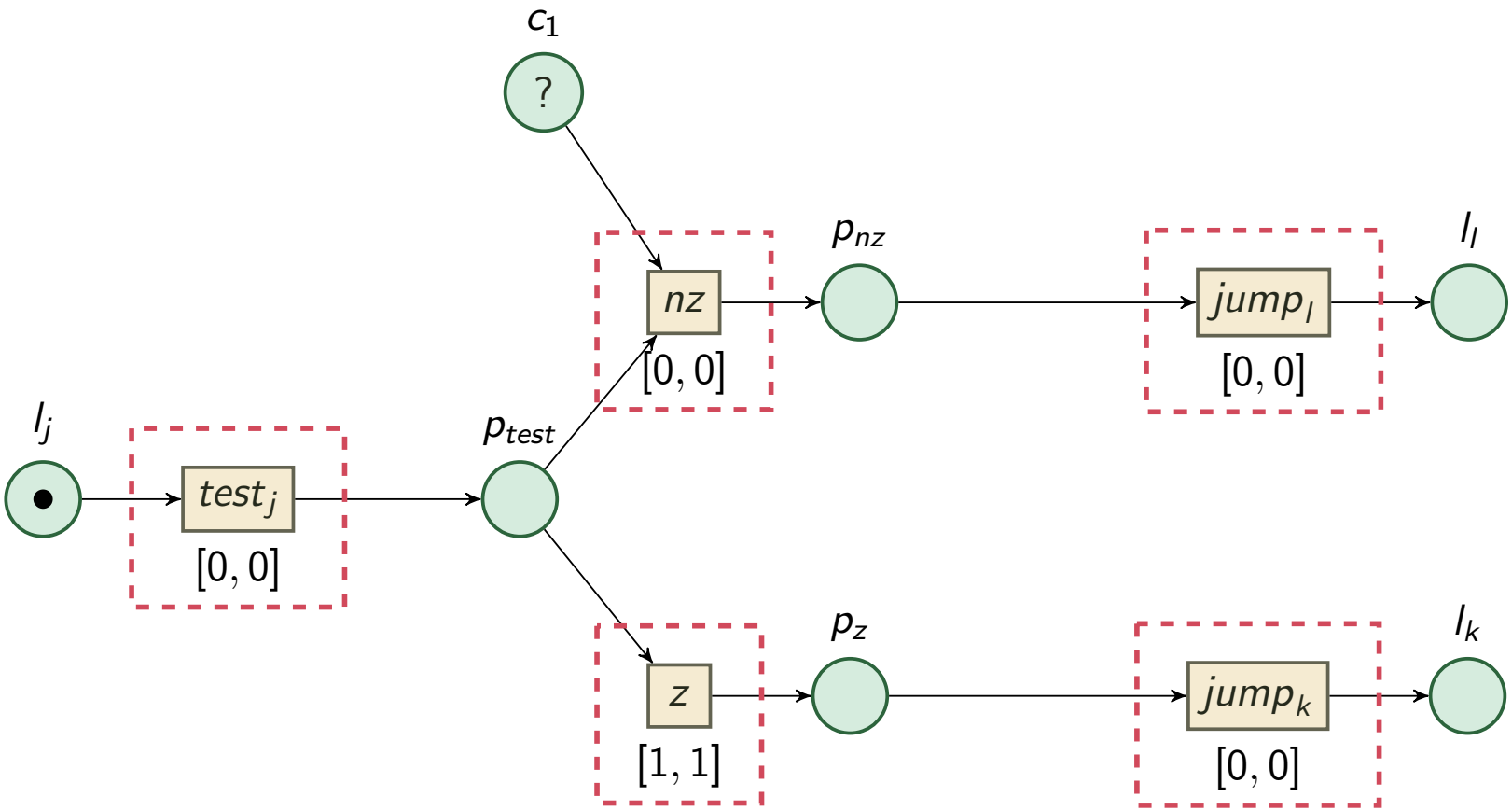
Reduce the halting problem to our problems of interest

2-counter machine with strong TPN[1]: increment



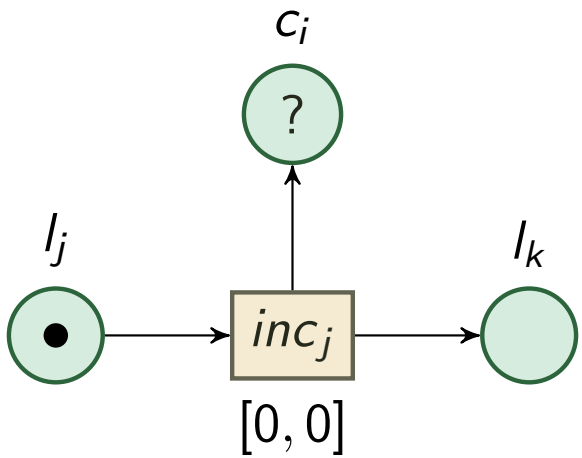
[1] N. D. Jones, L. H. Landweber, and Y. E. Lien. "Complexity of Some Problems in Petri Nets.". In: *Theoretical Computer Science* (1977).

2-counter machine with strong TPN[2]: zero-test

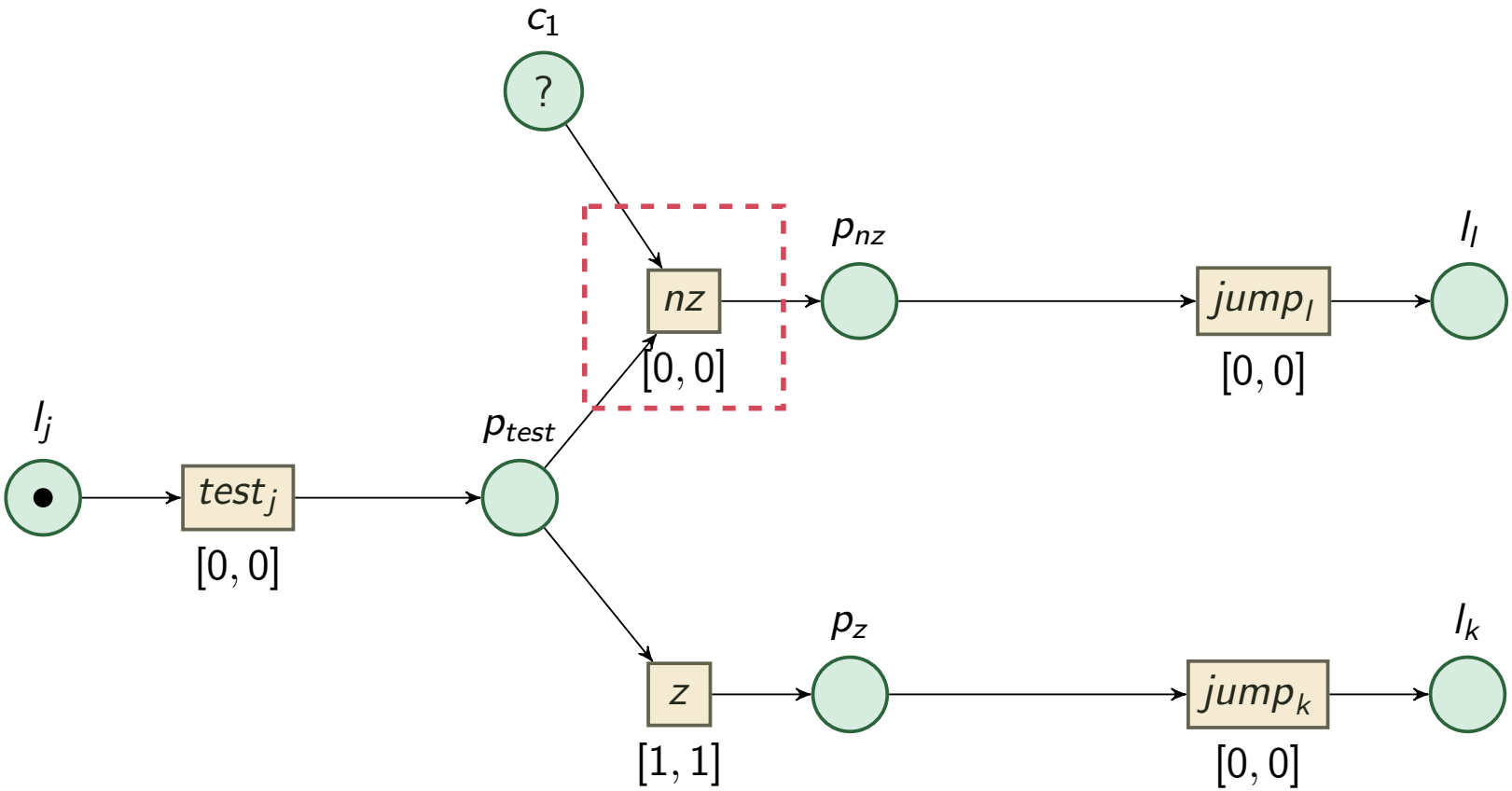


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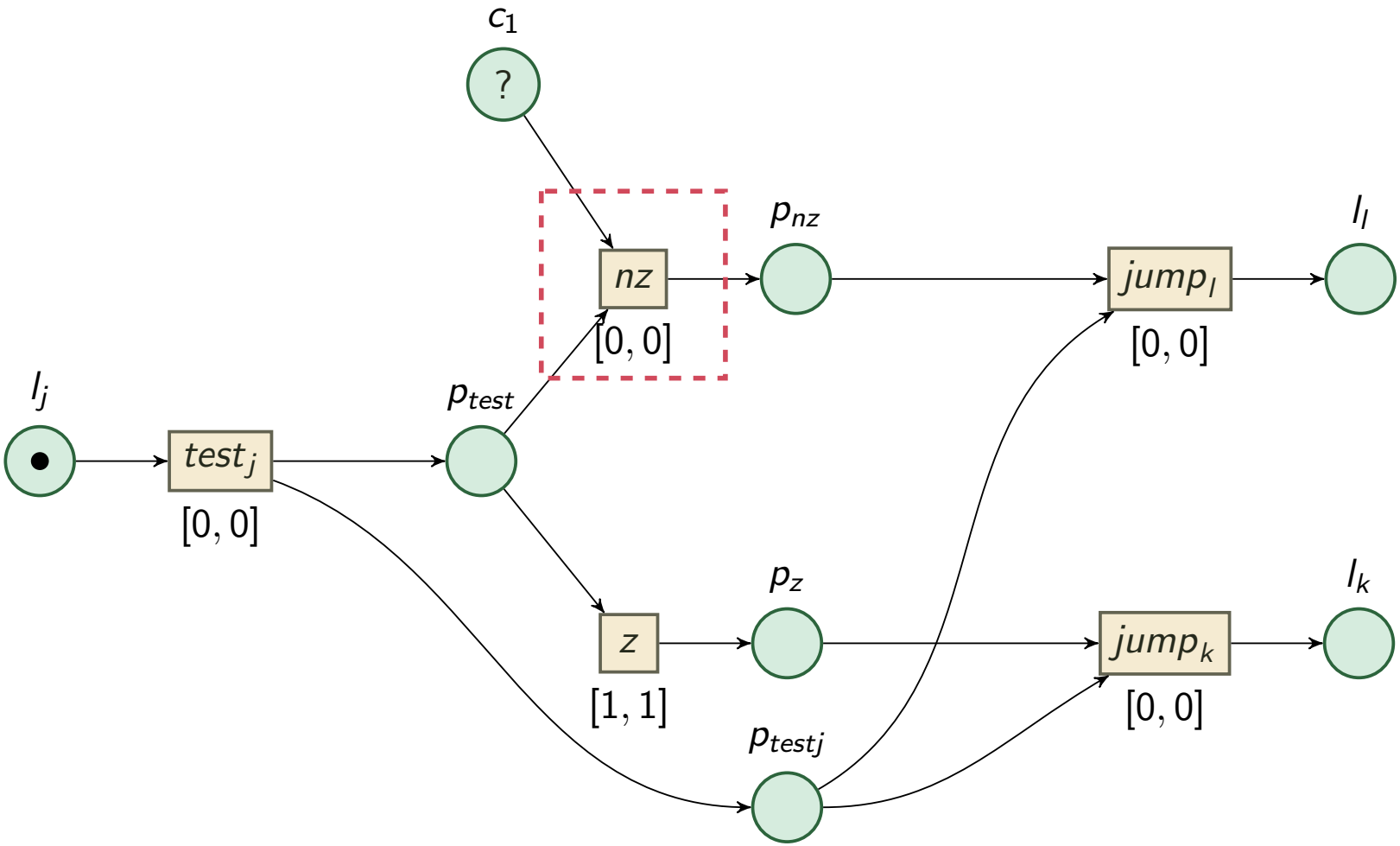
What is the minimum number of strong transitions that leads to undecidability?



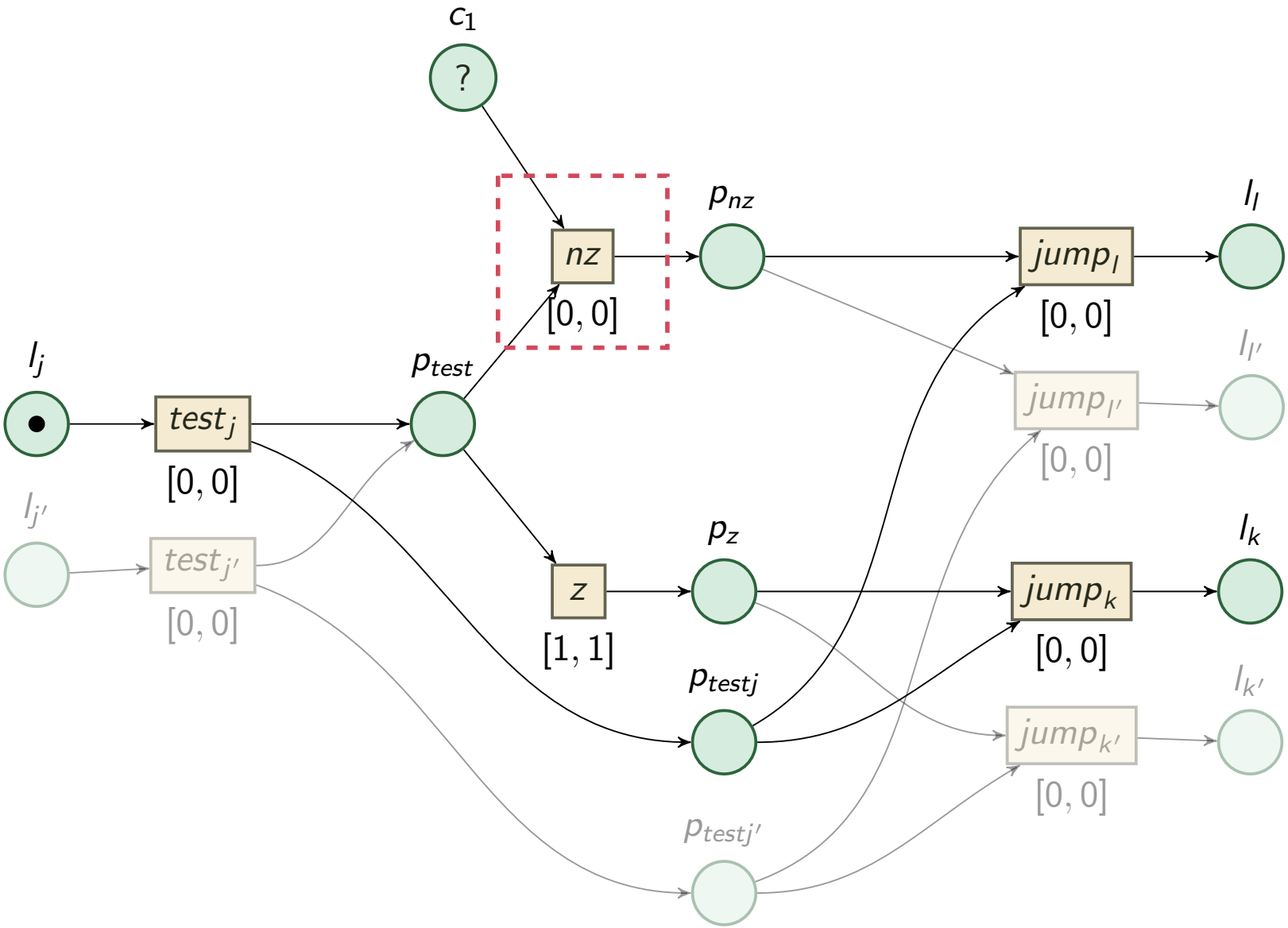
With one strong transition per jzdec instruction



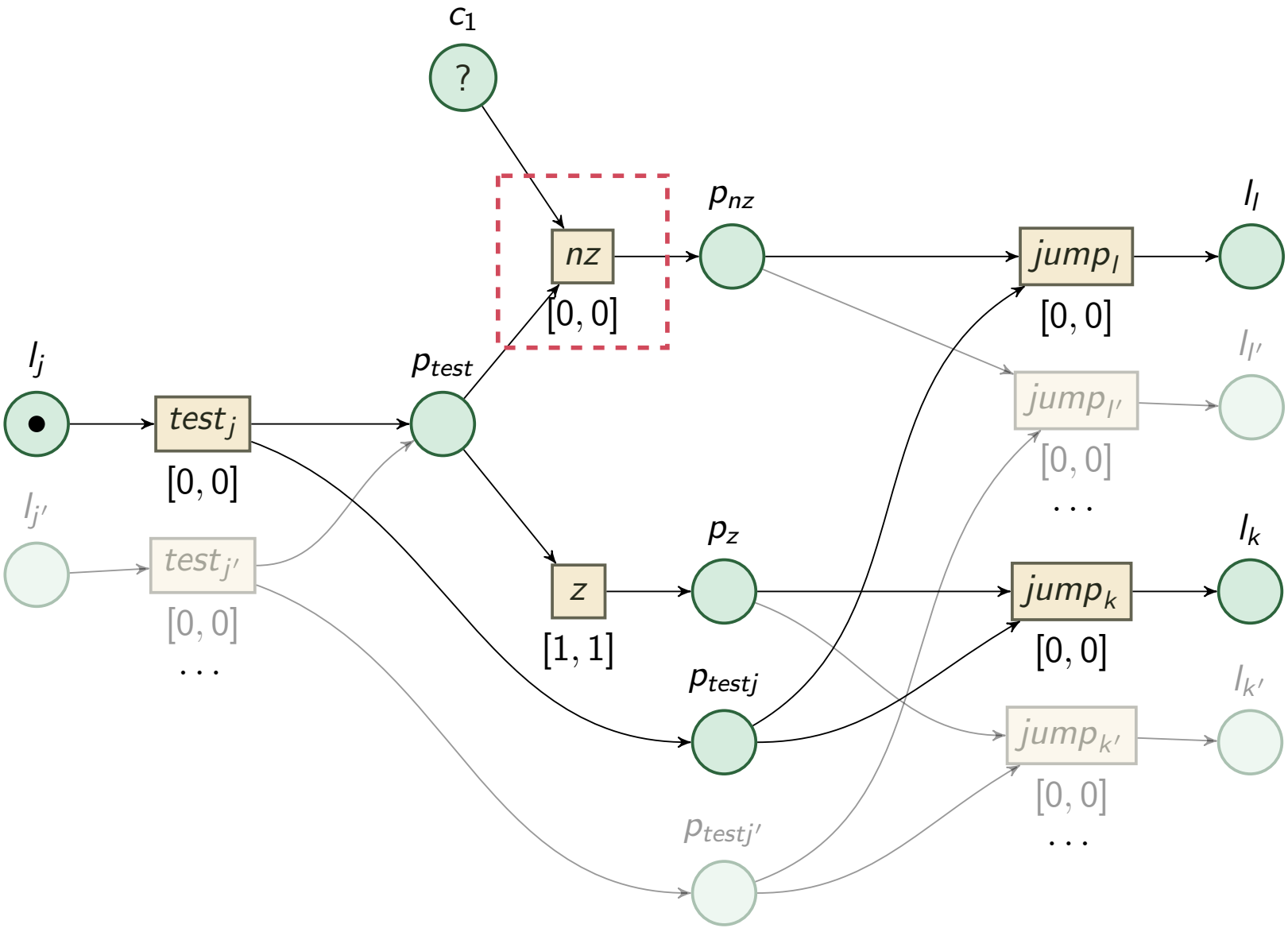
With one strong transition per counter



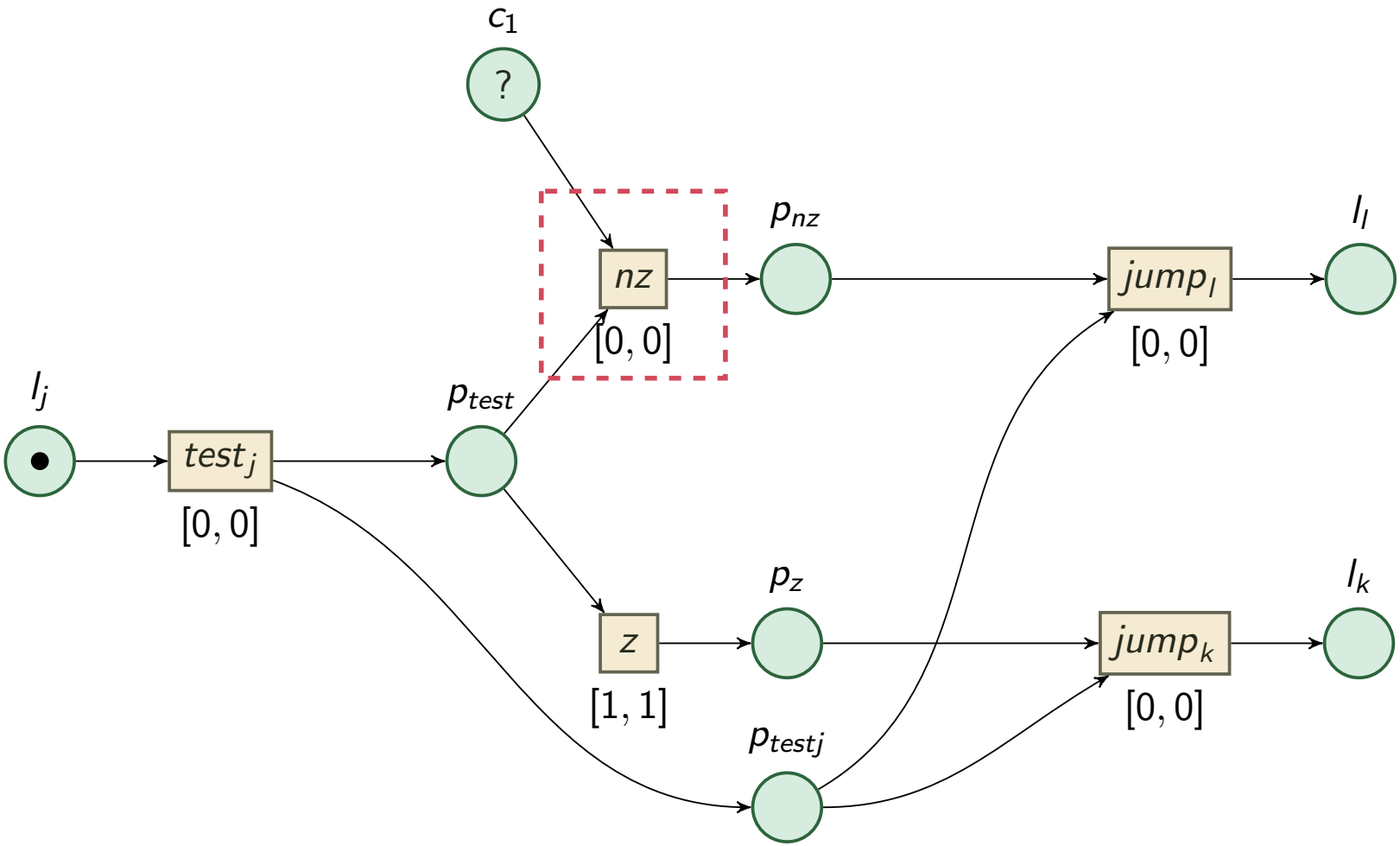
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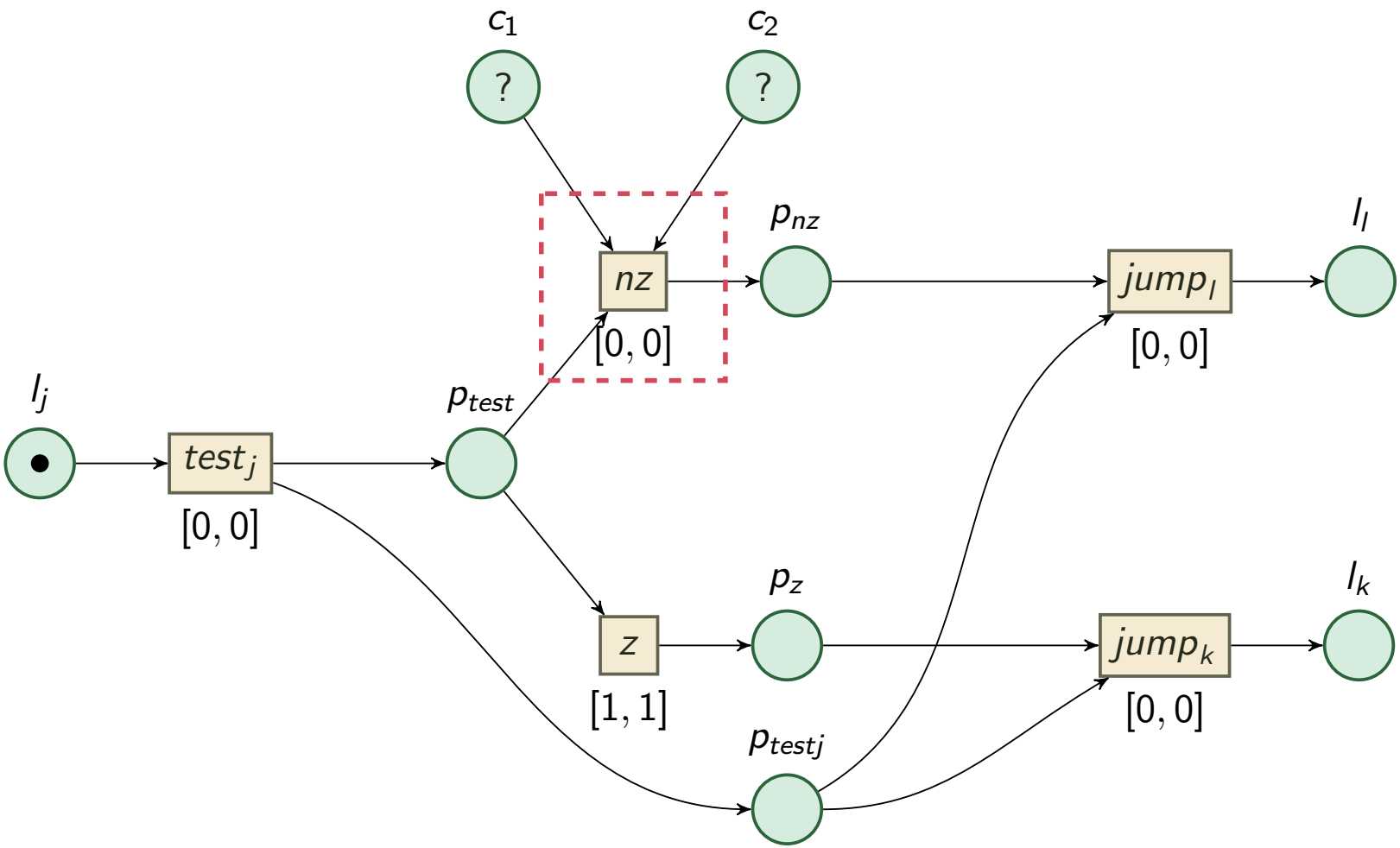
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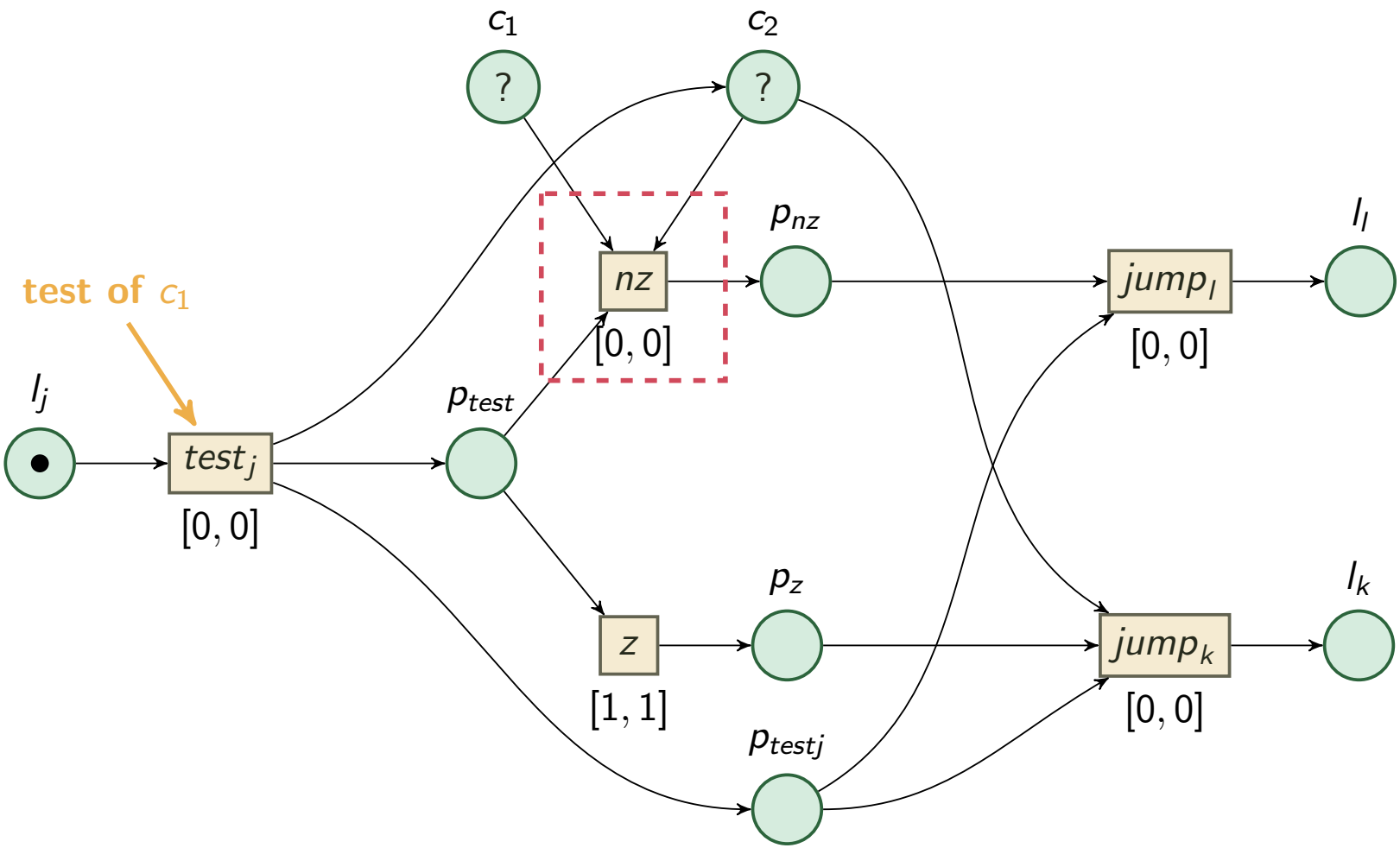
With a single strong transition



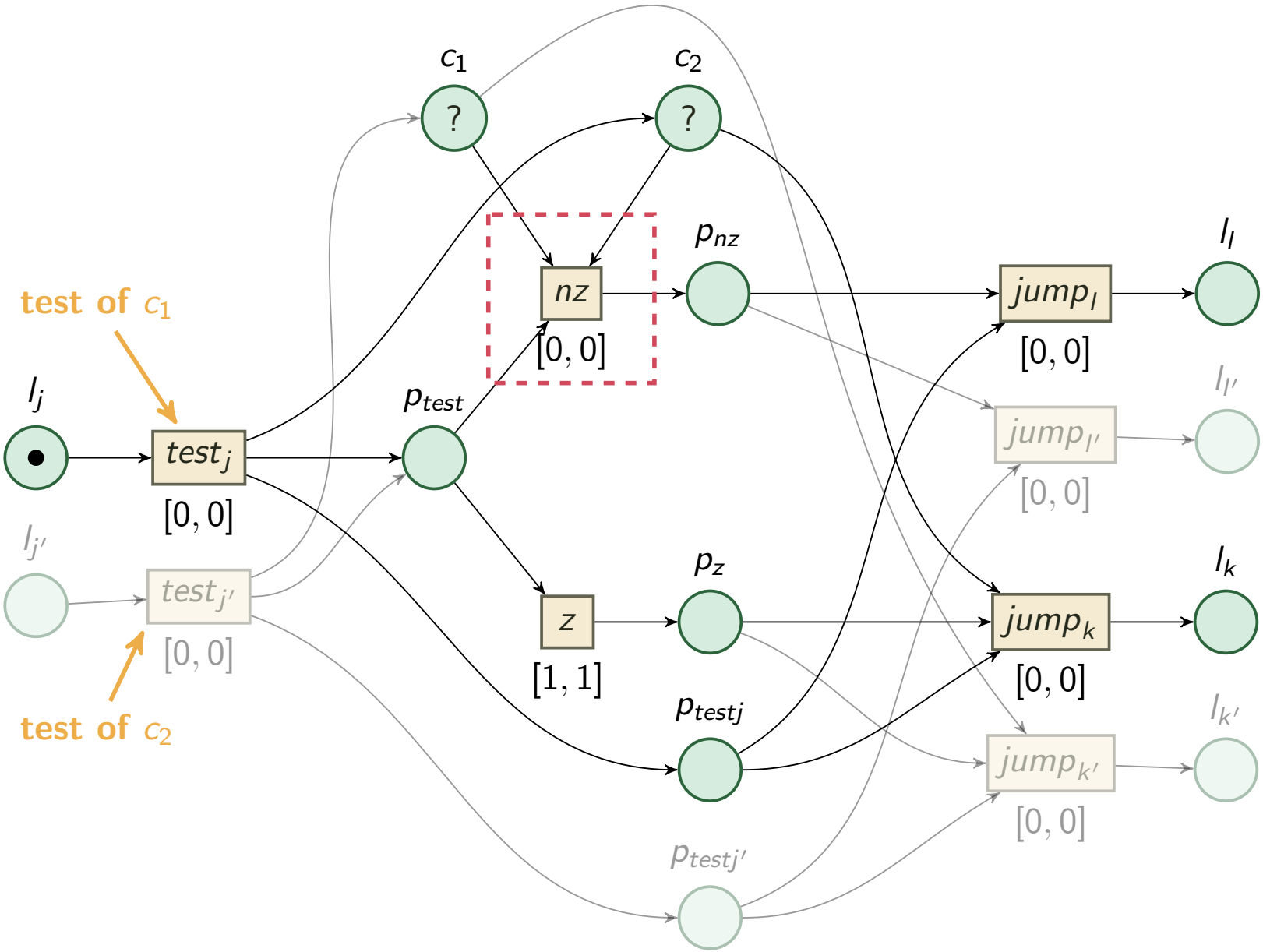
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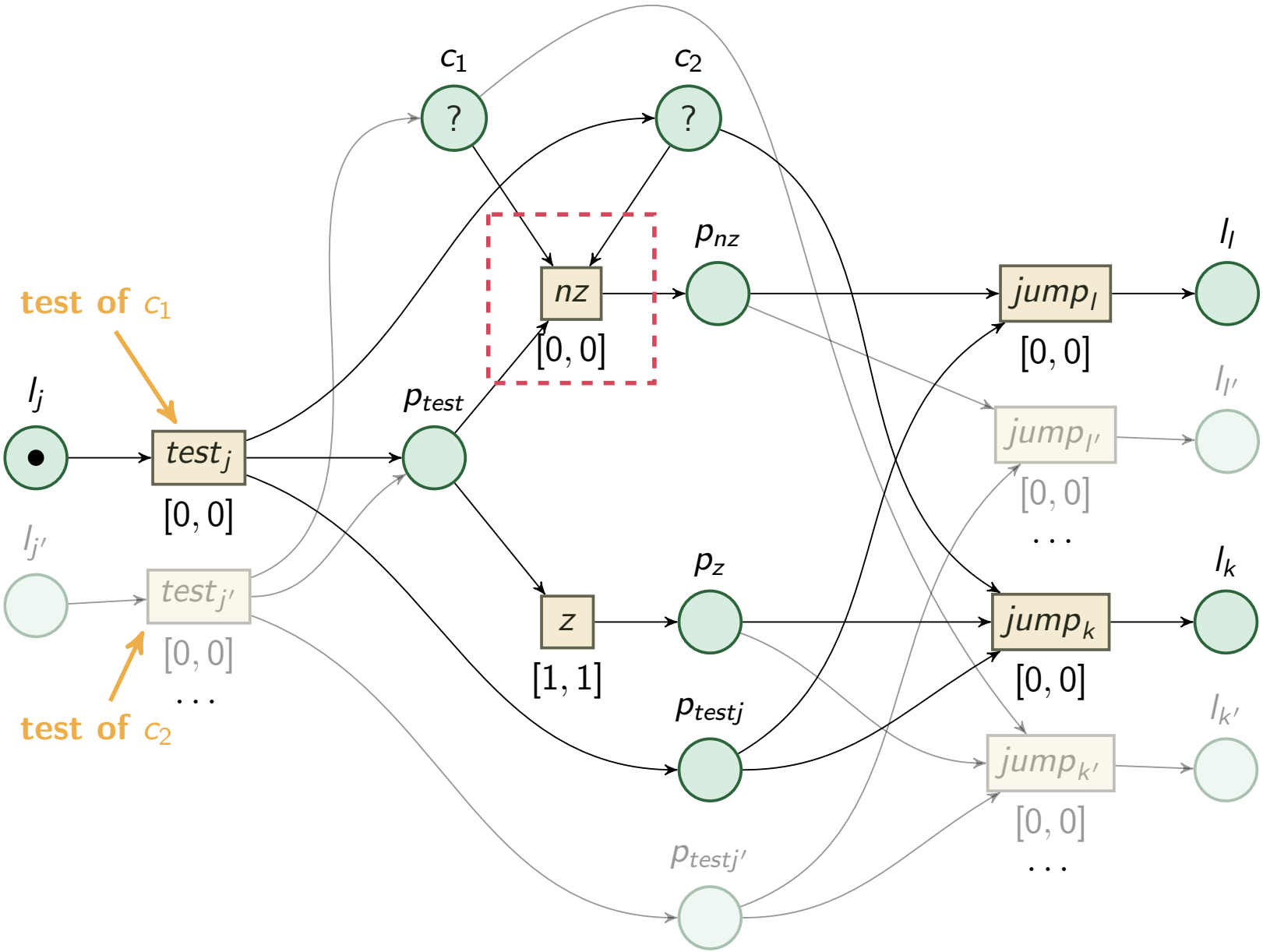
With a single strong transition



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With a single strong transition



Outline

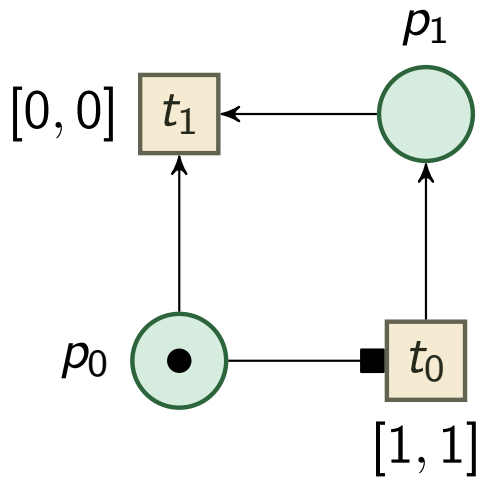
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Weakly-strong TPN

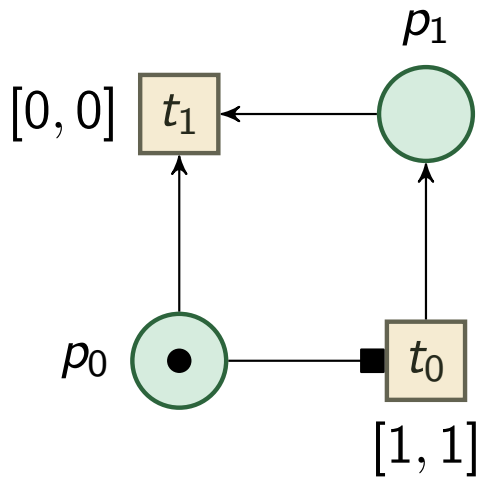
Weak TPN with read, reset or transfer arcs

Conclusion

Read arc

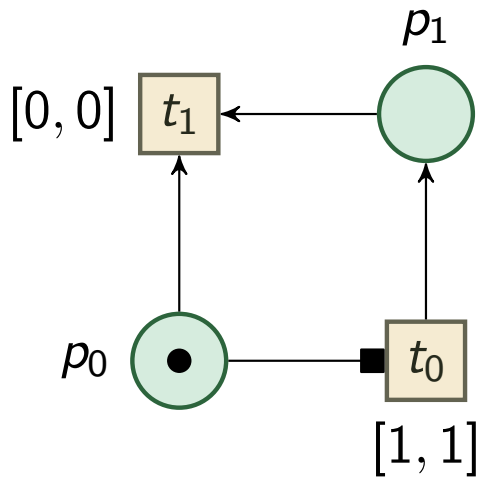


Read arc



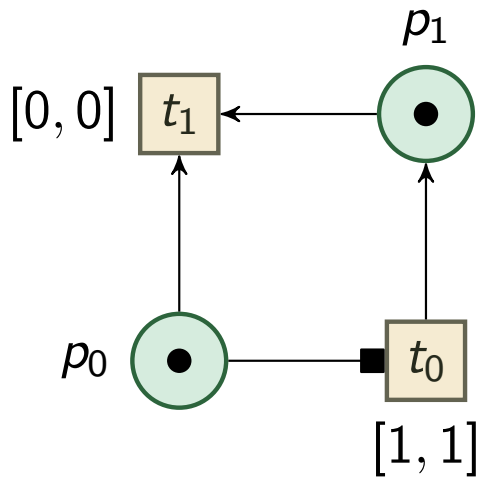
$t_0 \in [1,1]$
$t_1 \in -$

Read arc



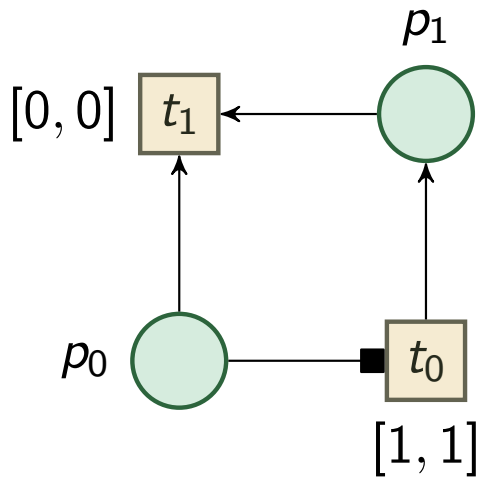
$$\begin{array}{lcl} t_0 \in [1,1] & \xrightarrow{1} & [0,0] \\ t_1 \in - & & - \end{array}$$

Read arc



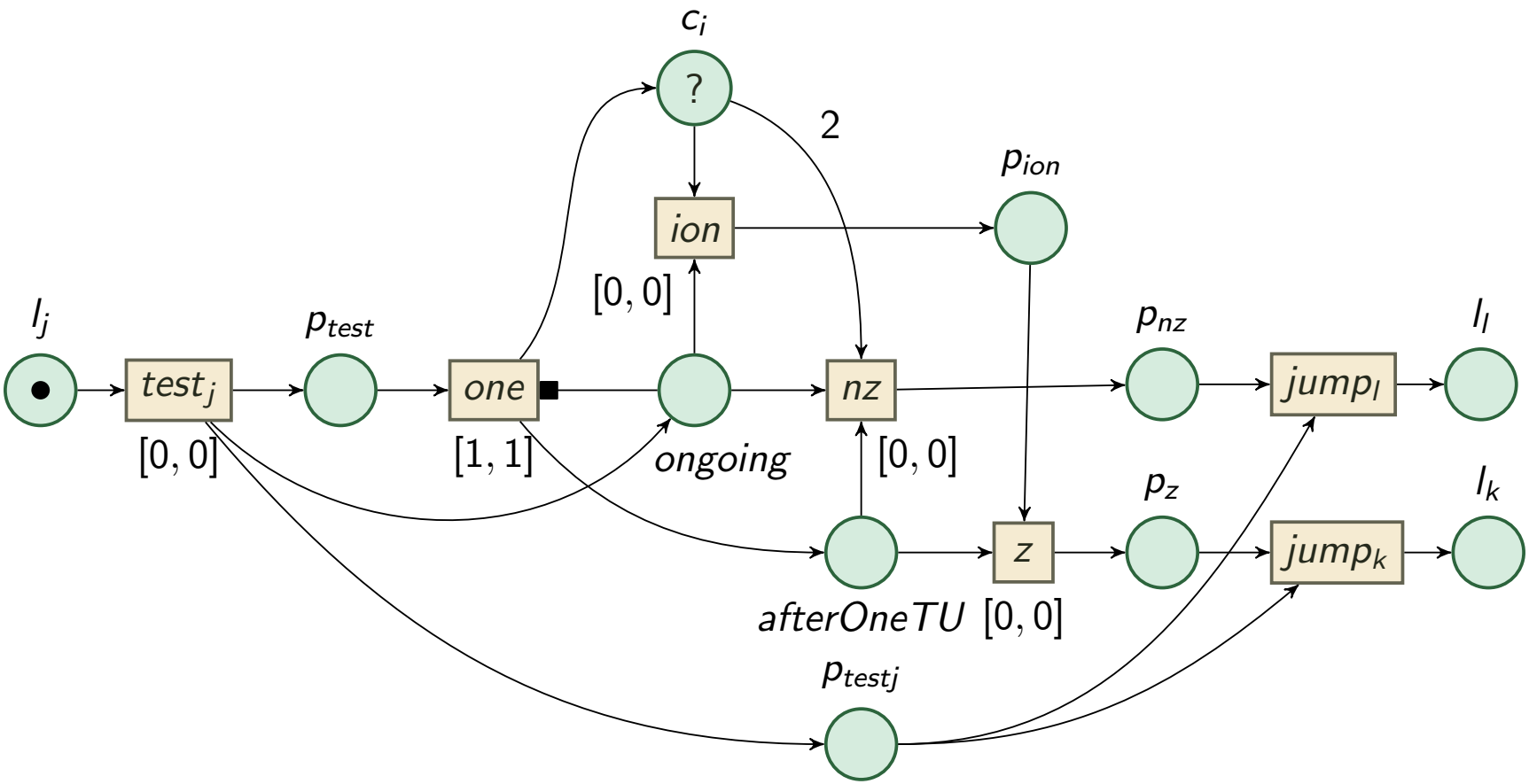
$$\begin{array}{l} t_0 \in [1,1] \\ t_1 \in - \end{array} \xrightarrow{1} \begin{array}{l} [0,0] \\ - \end{array} \xrightarrow{t_0} \begin{array}{l} [1,1] \\ [0,0] \end{array}$$

Read arc

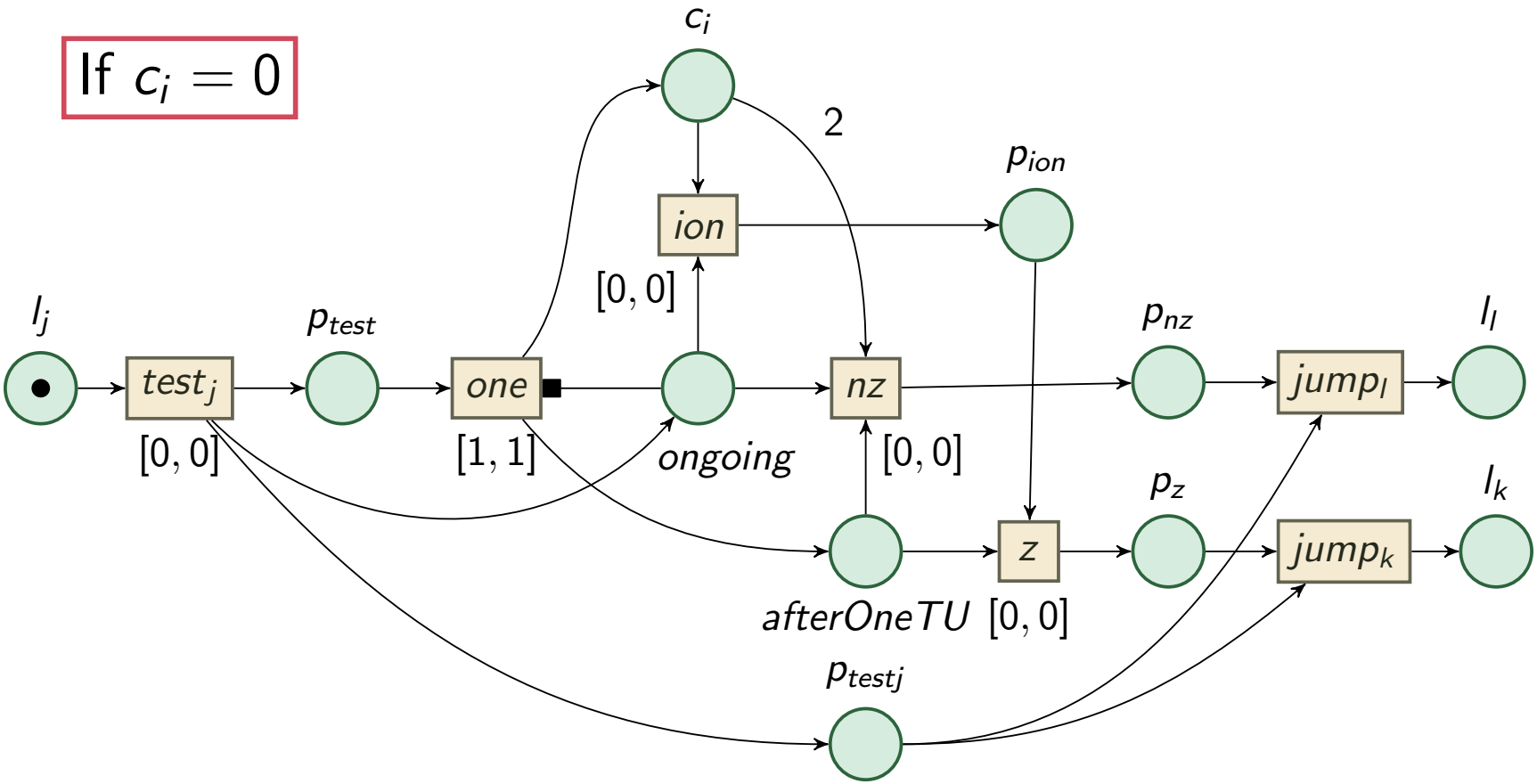


$$\begin{array}{lcccccc} t_0 \in [1,1] & \xrightarrow{1} & [0,0] & \xrightarrow{t_0} & [1,1] & \xrightarrow{t_1} & - \\ t_1 \in - & & - & & [0,0] & & - \end{array}$$

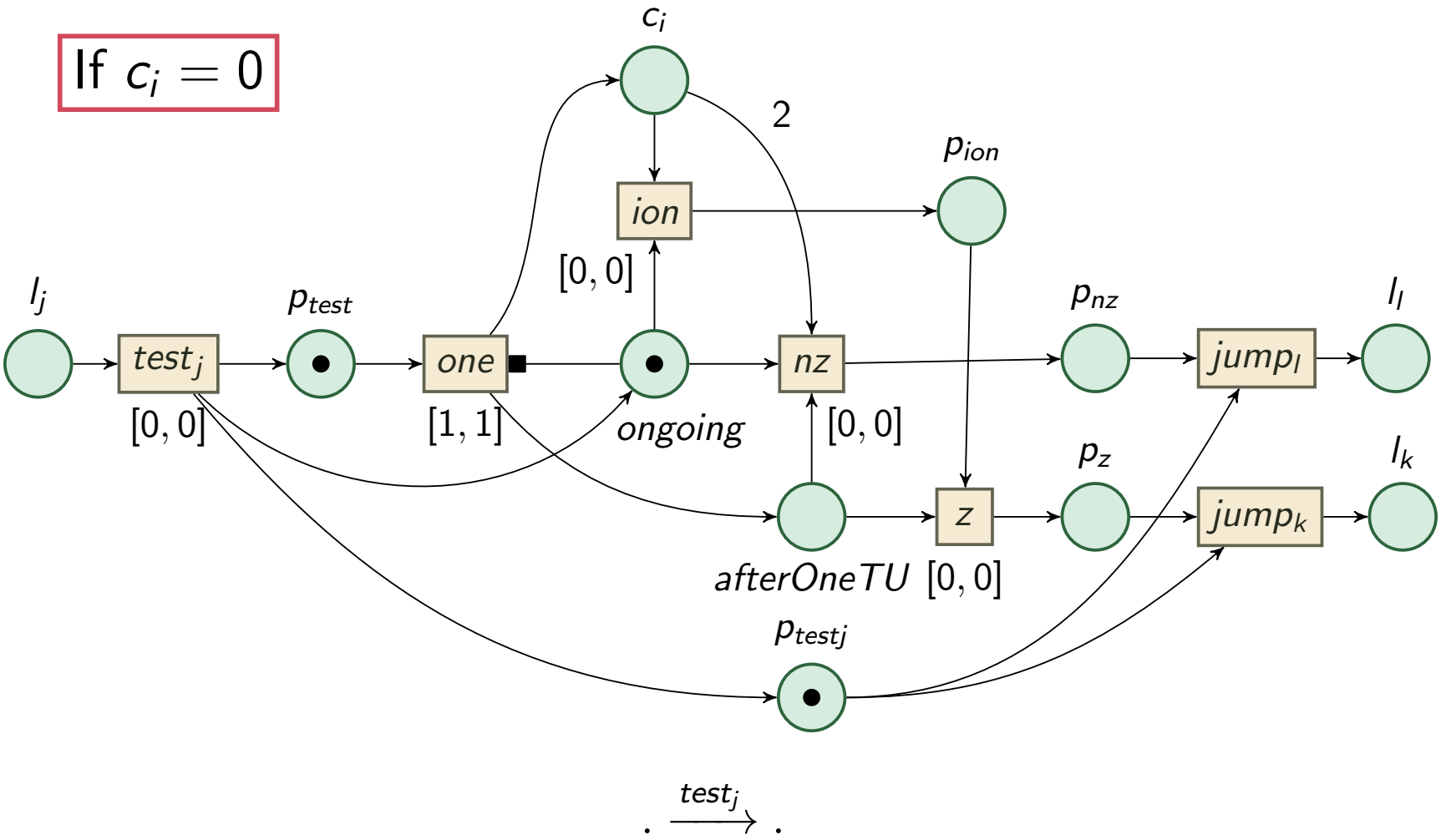
Zero-test with one read arcs



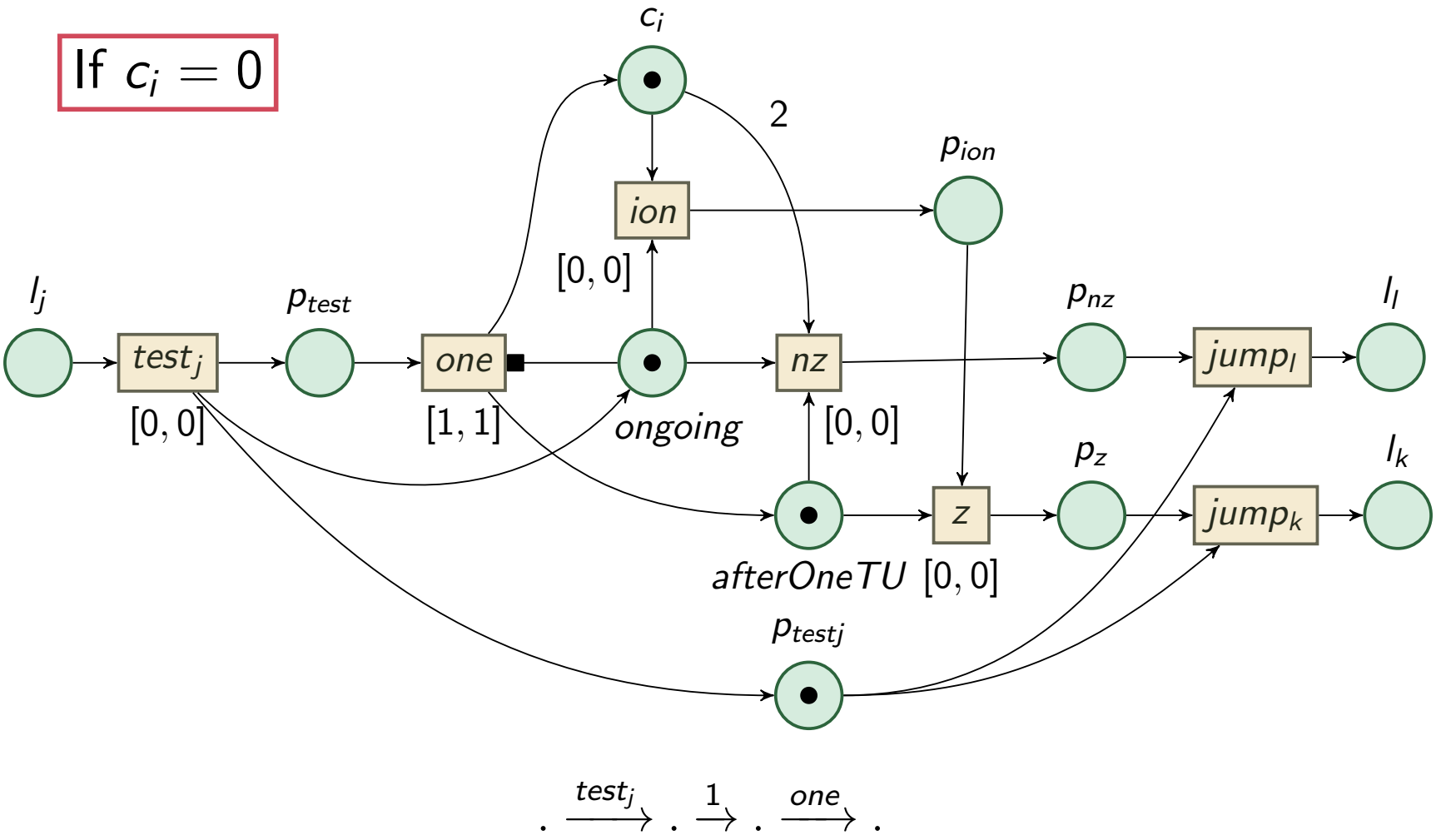
Zero-test with one read arcs



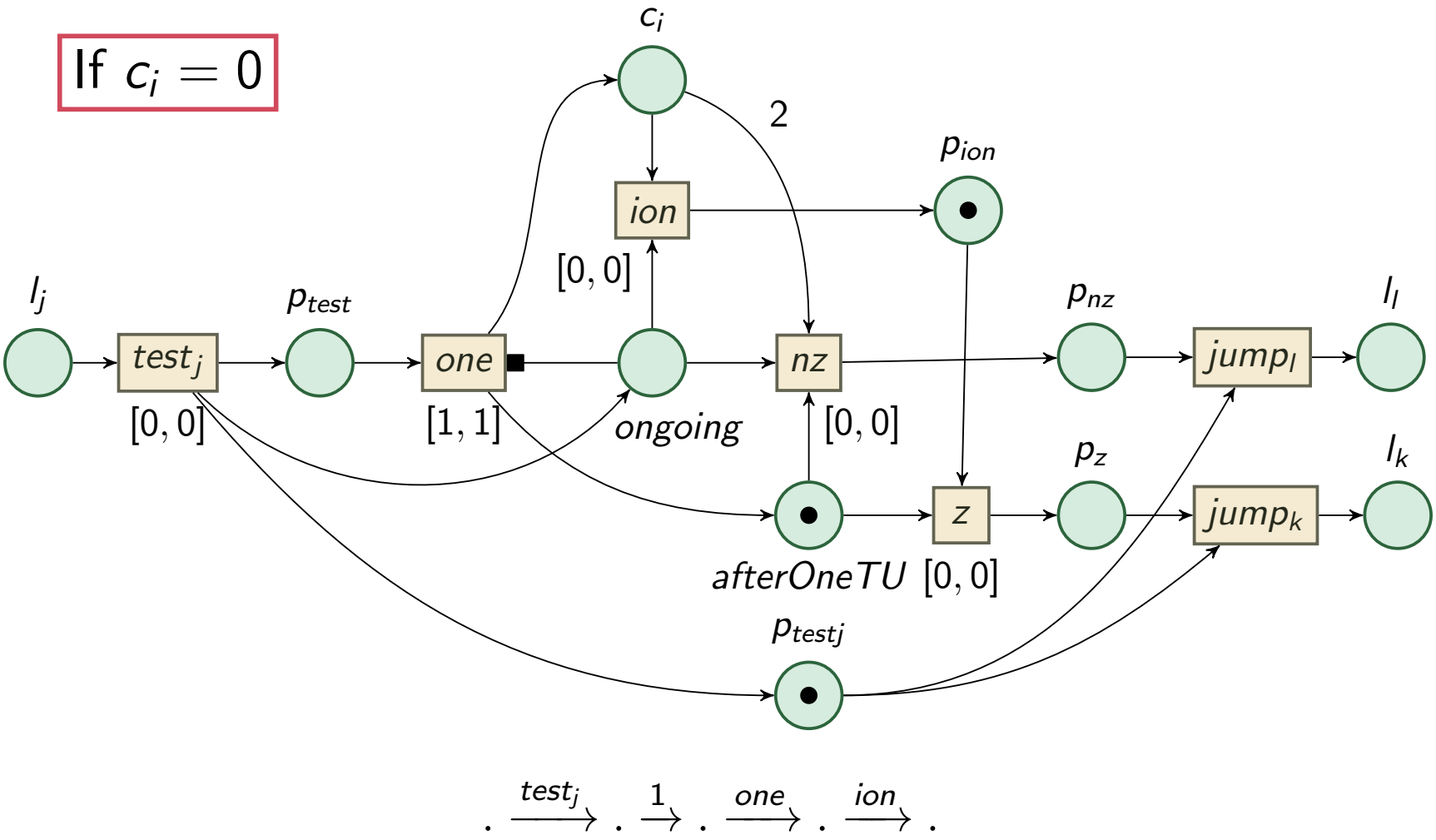
Zero-test with one read arcs



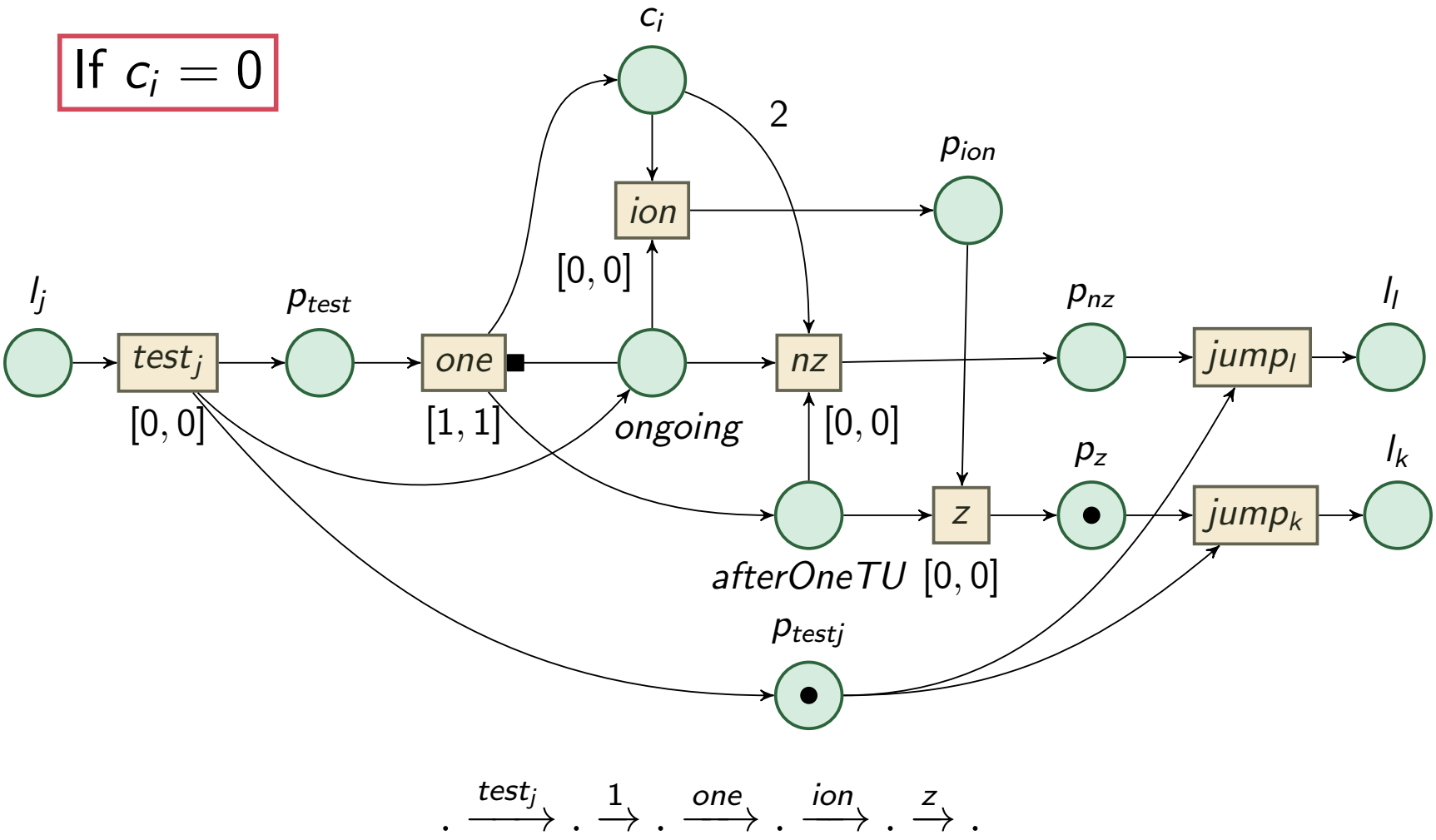
Zero-test with one read arcs



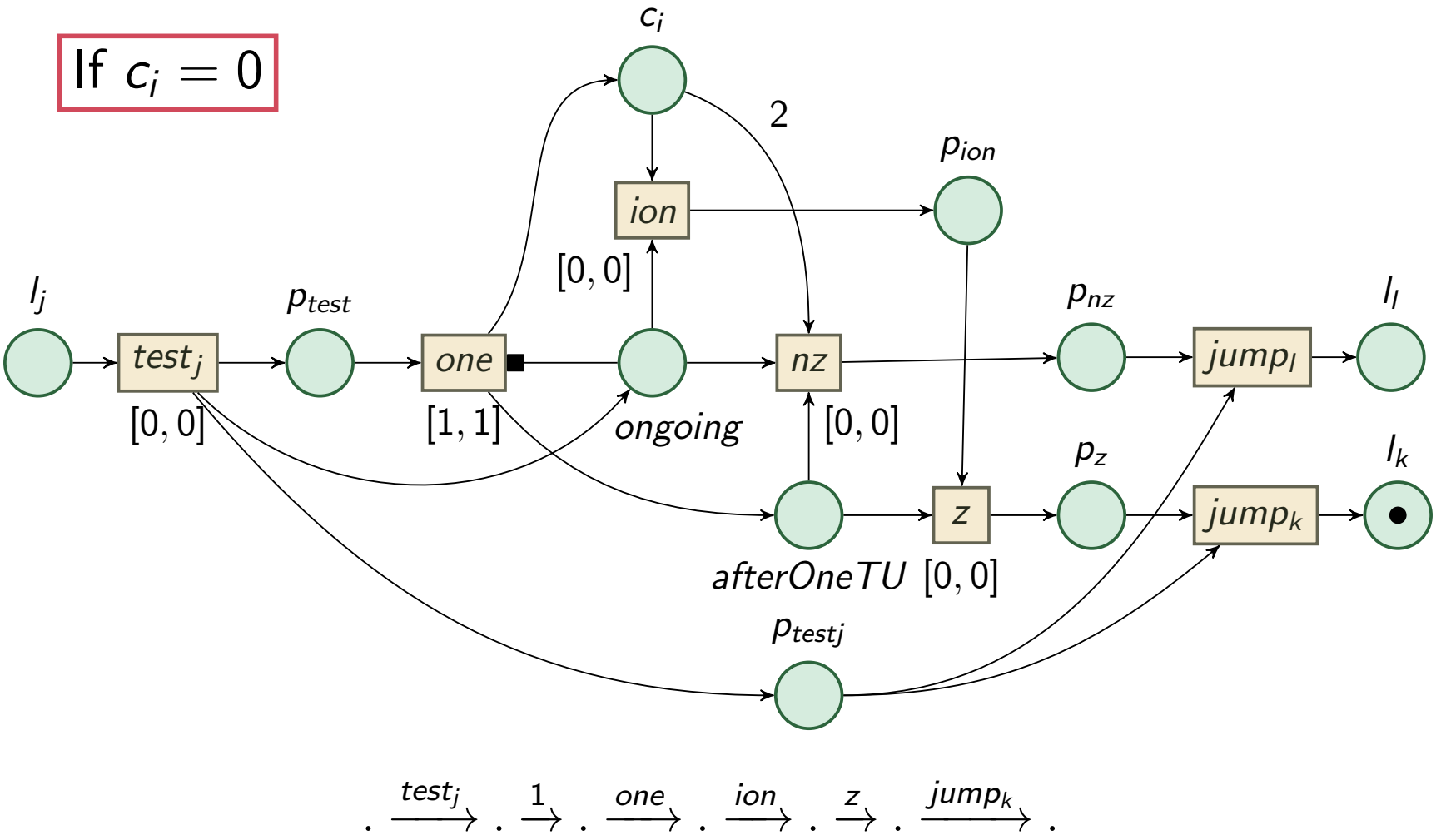
Zero-test with one read arcs



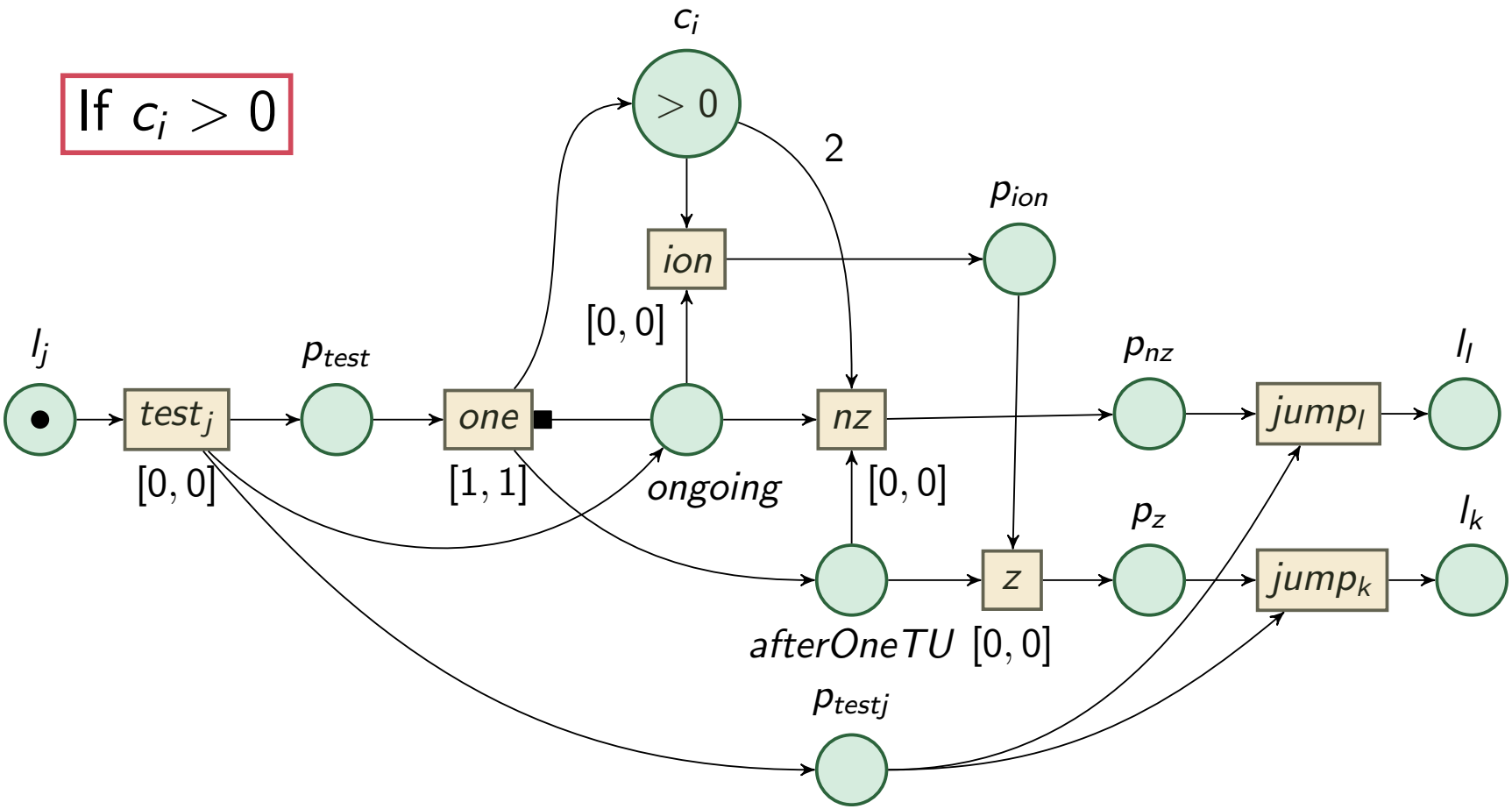
Zero-test with one read arcs



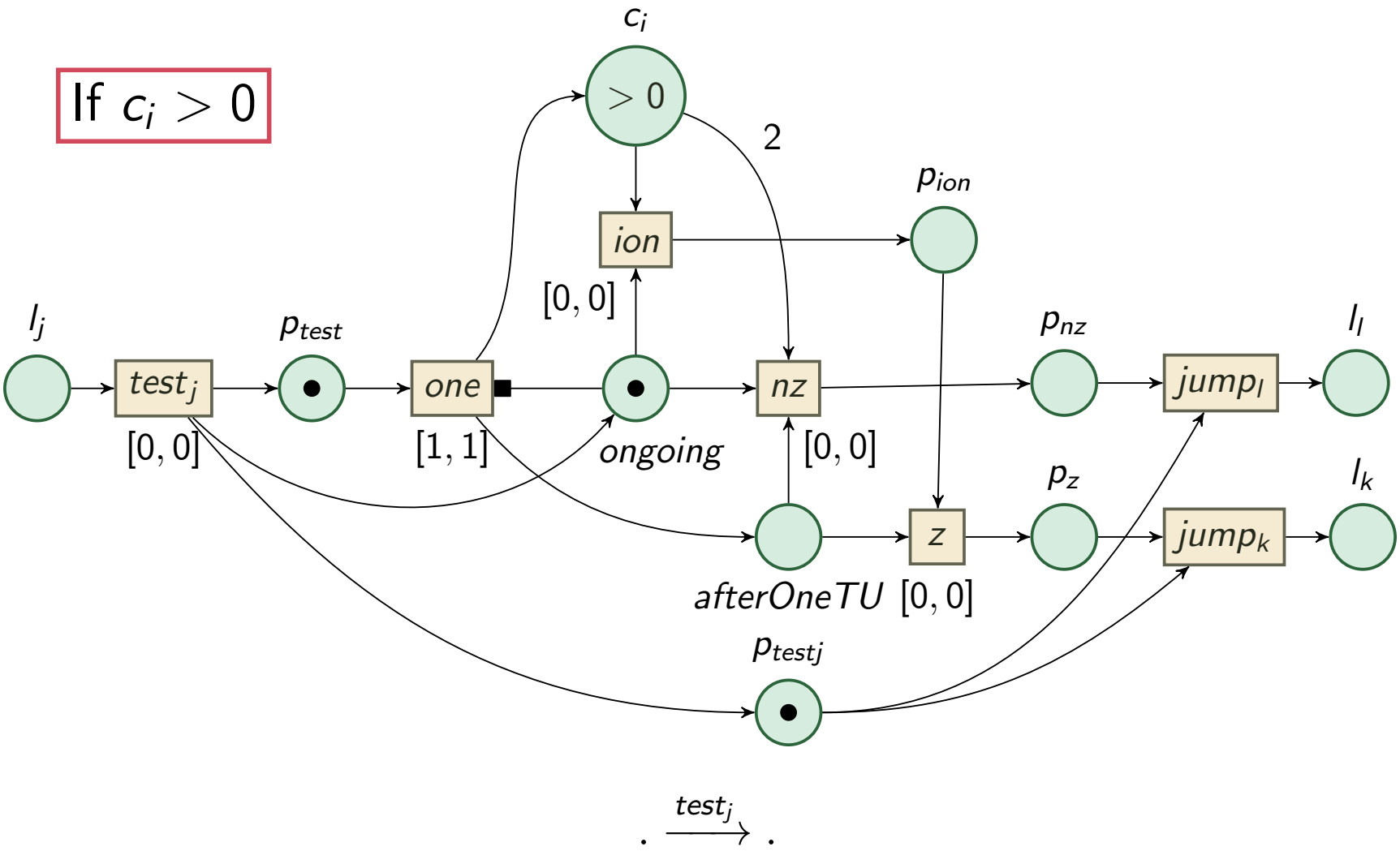
Zero-test with one read arcs



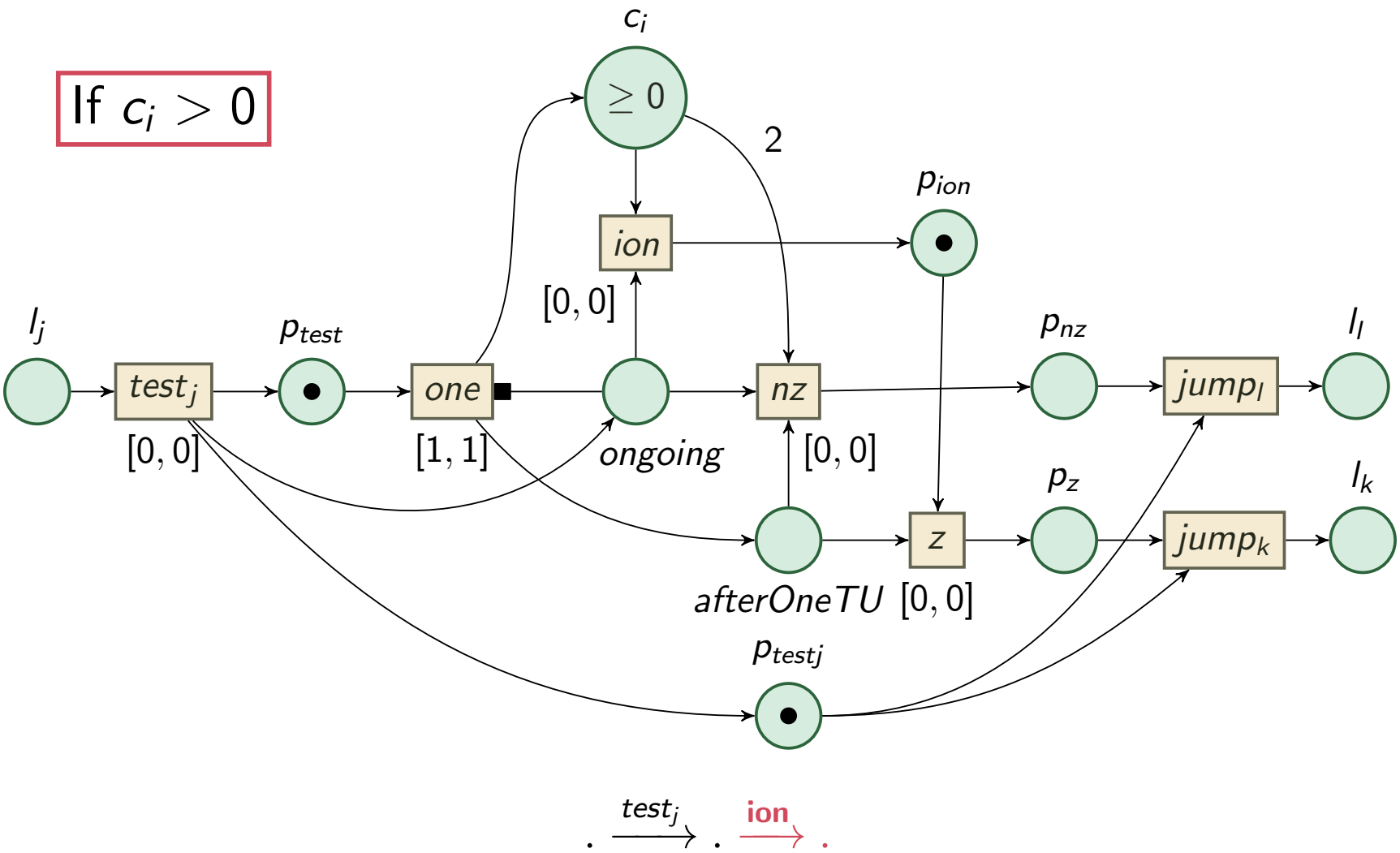
Zero-test with one read arcs



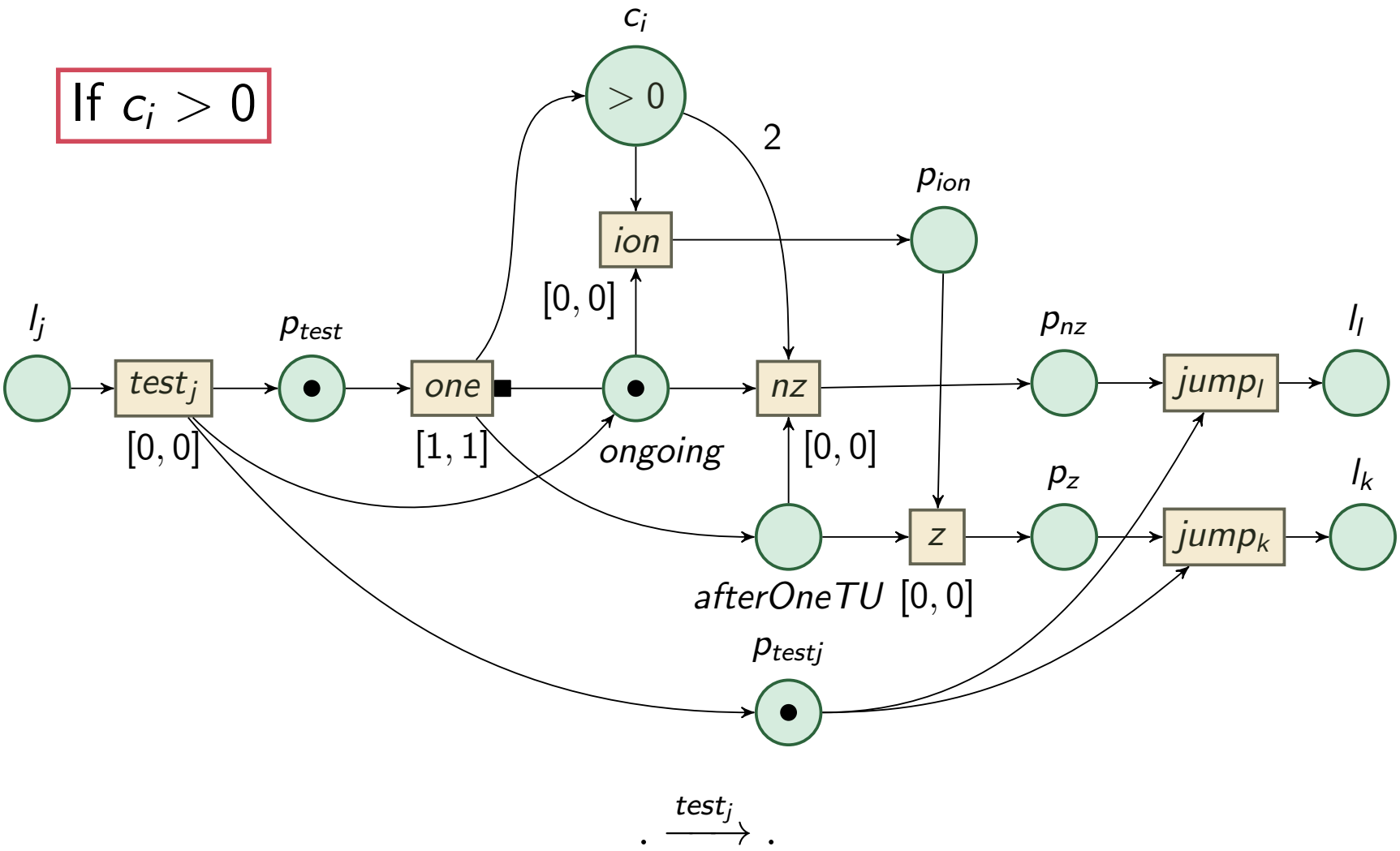
Zero-test with one read arcs



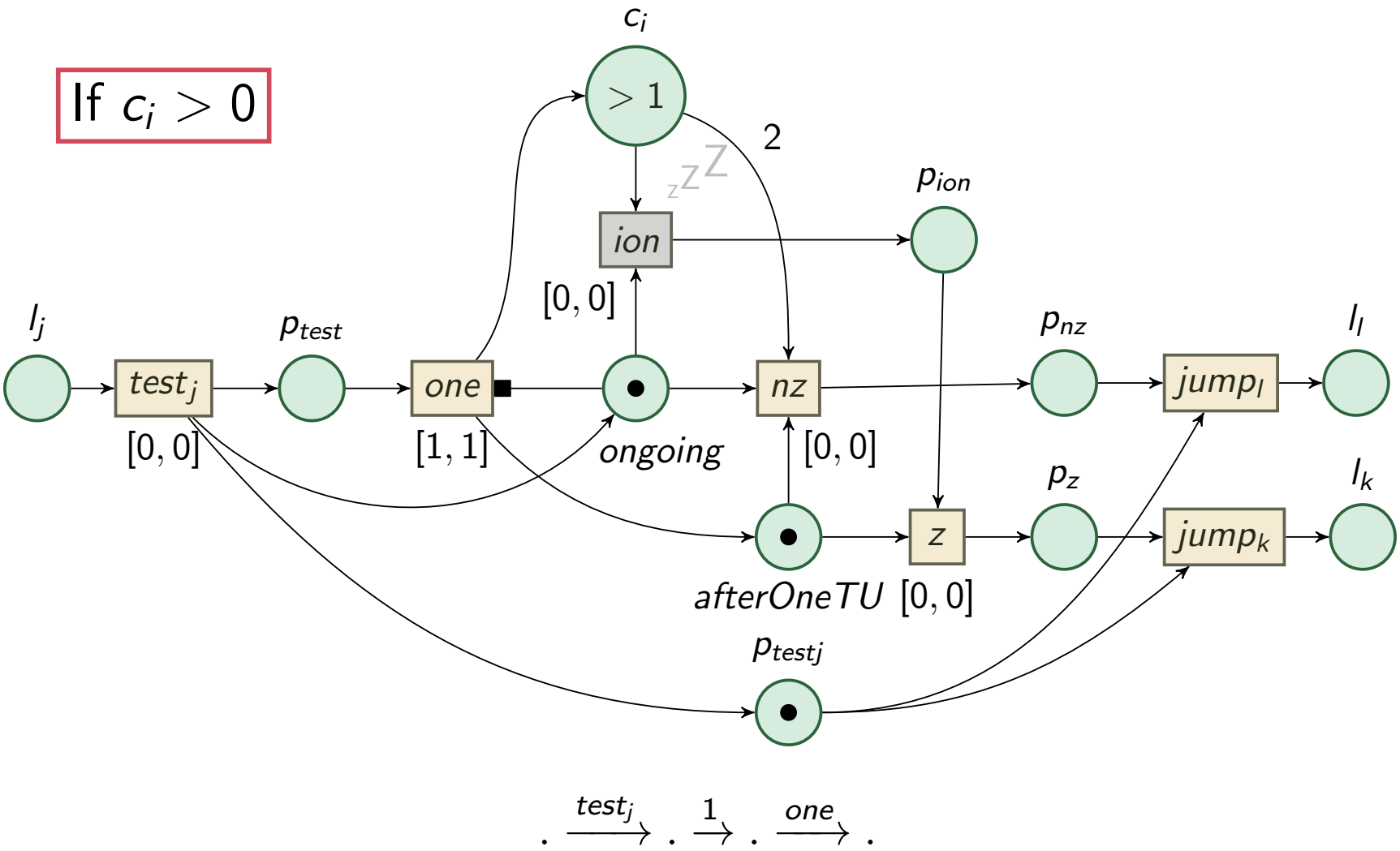
Zero-test with one read arcs



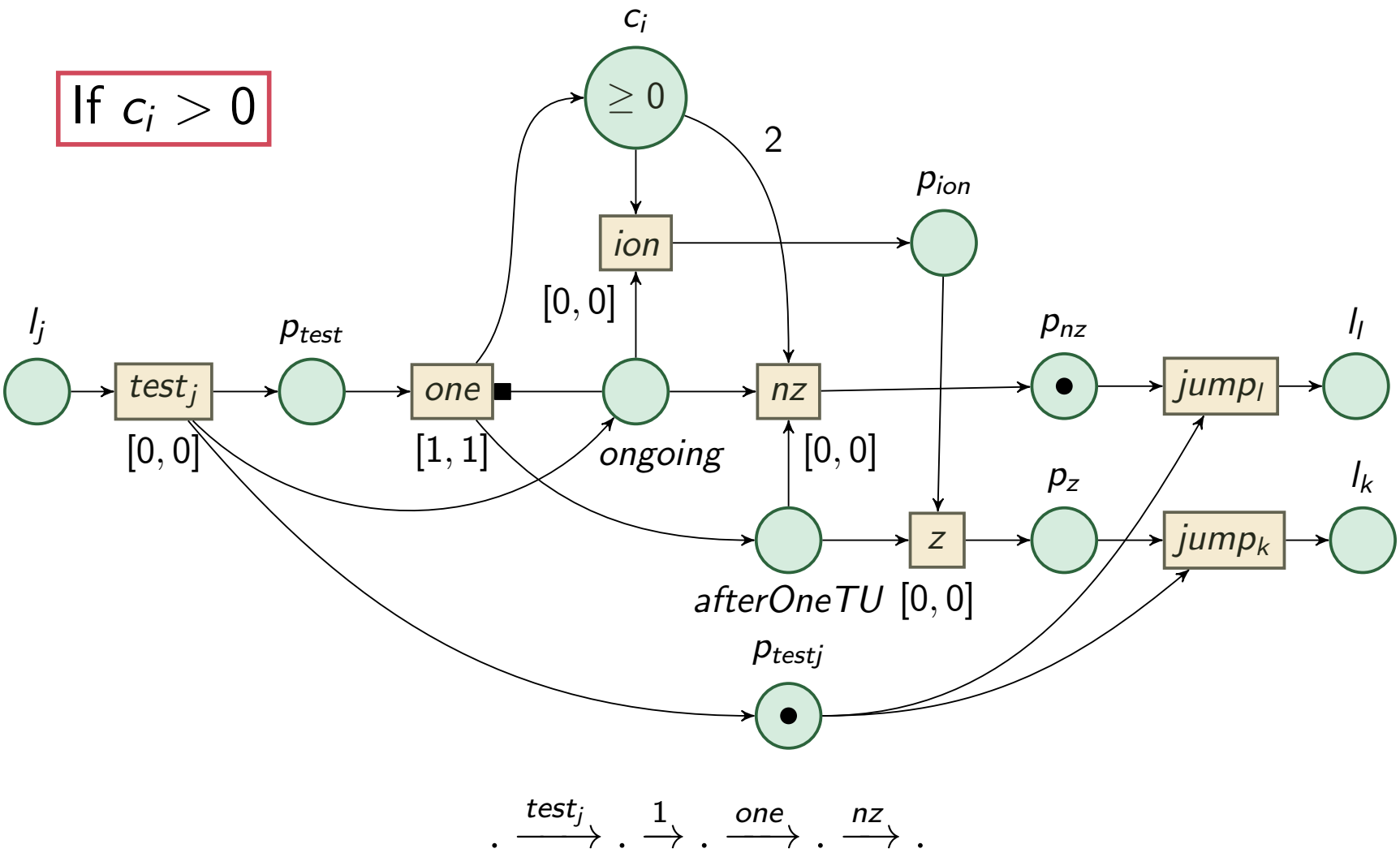
Zero-test with one read arcs



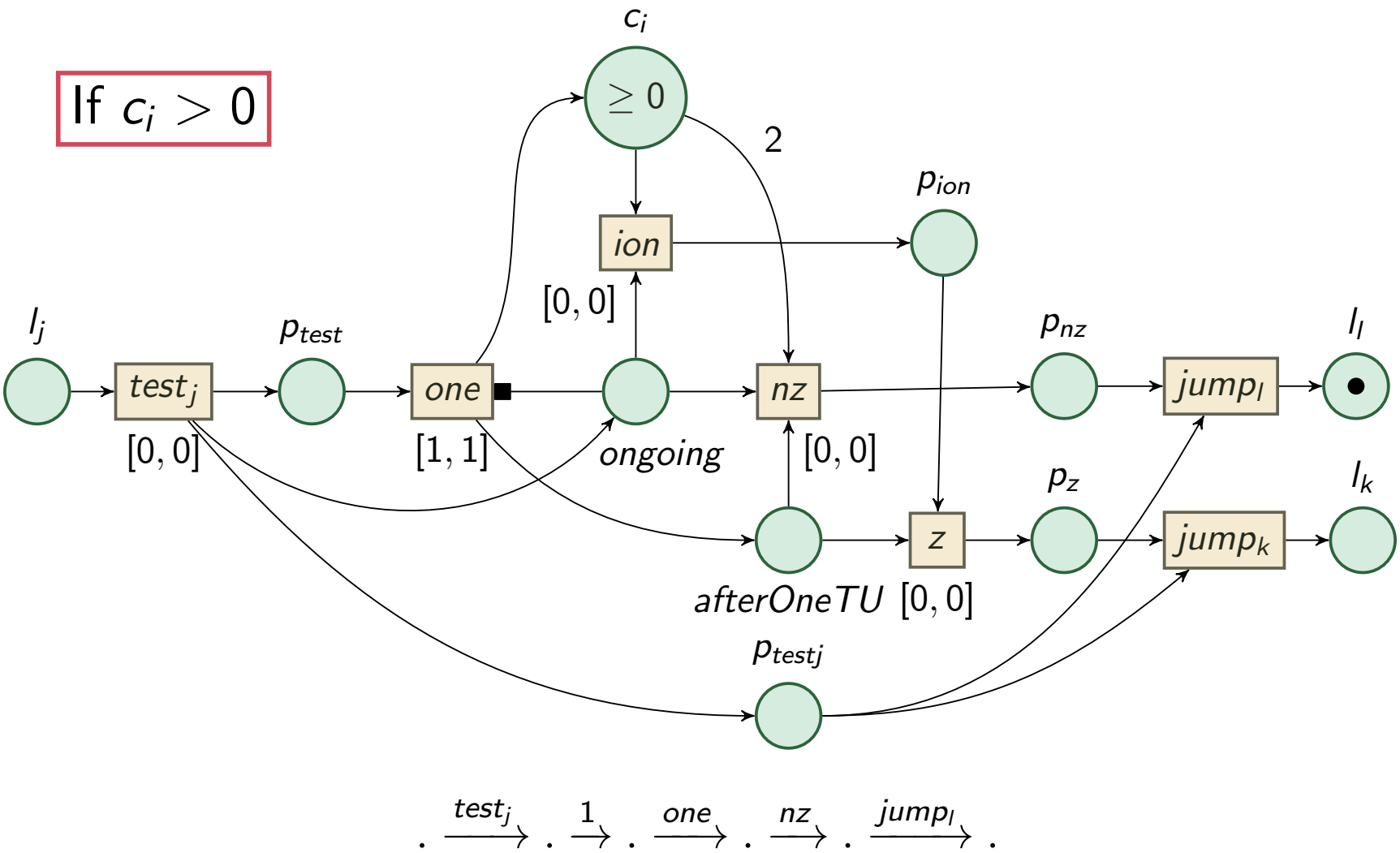
Zero-test with one read arcs



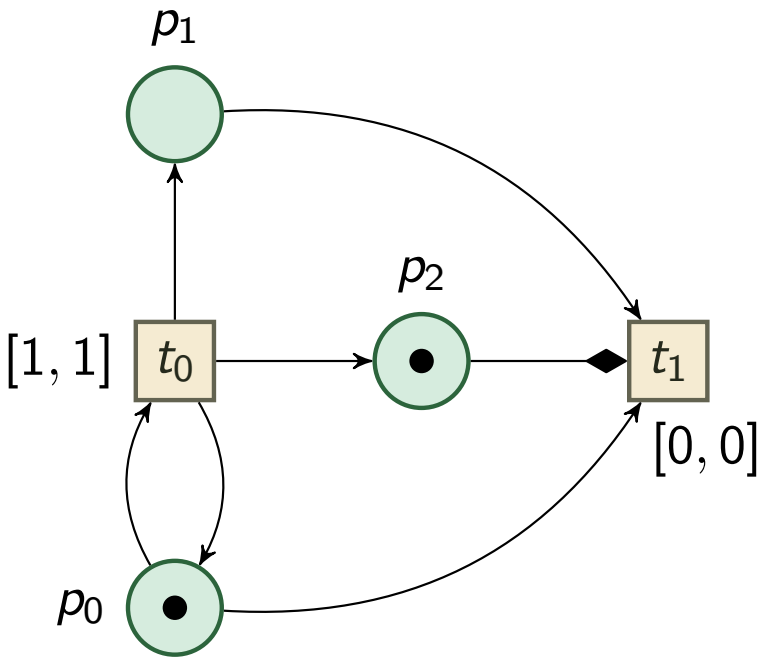
Zero-test with one read arcs



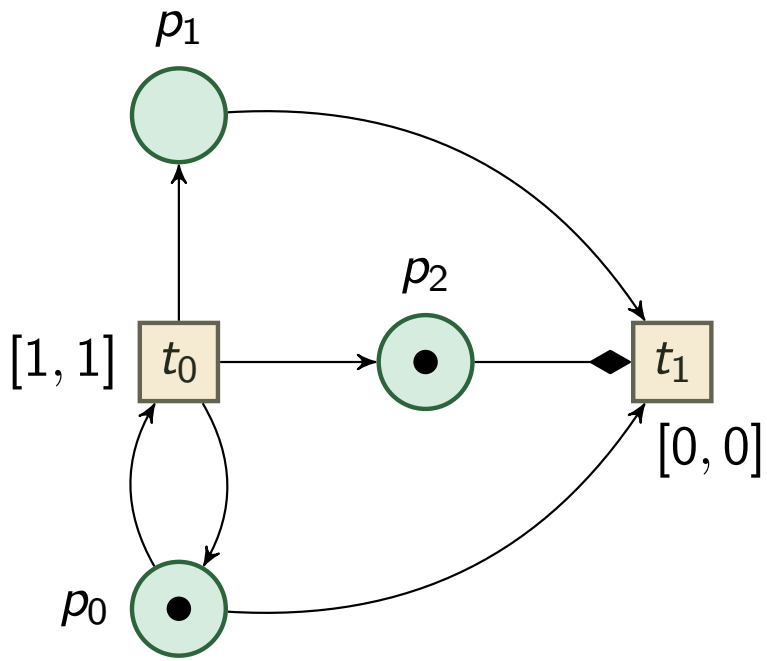
Zero-test with one read arcs



Reset and transfer arcs

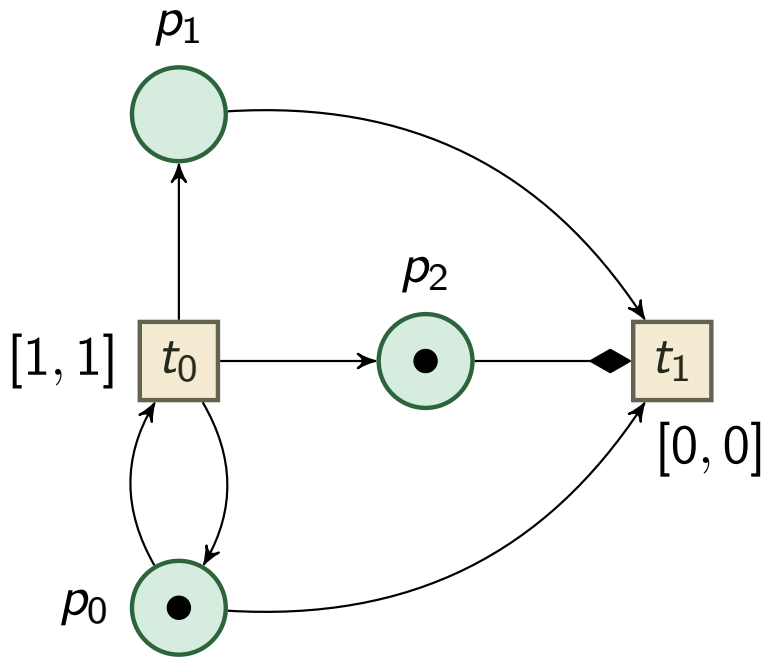


Reset and transfer arcs



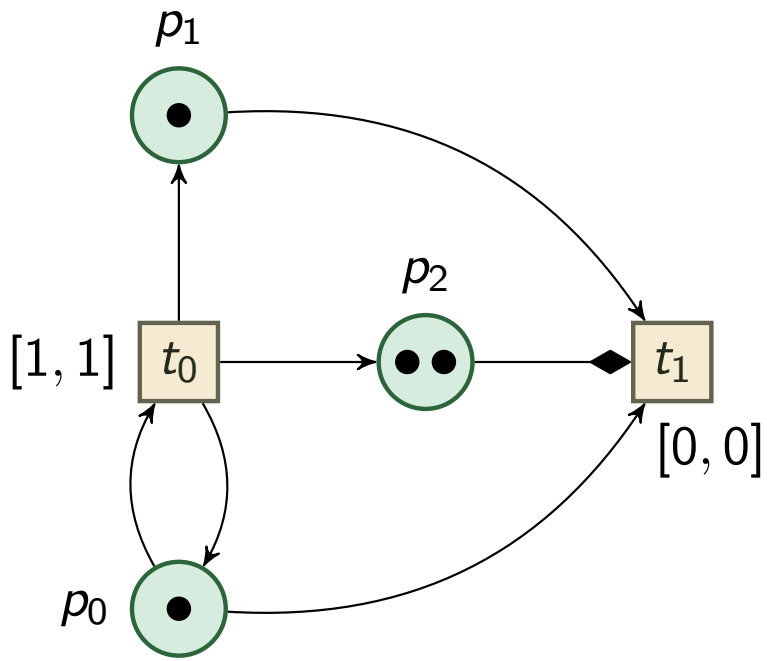
$t_0 \in [1,1]$
$t_1 \in -$

Reset and transfer arcs



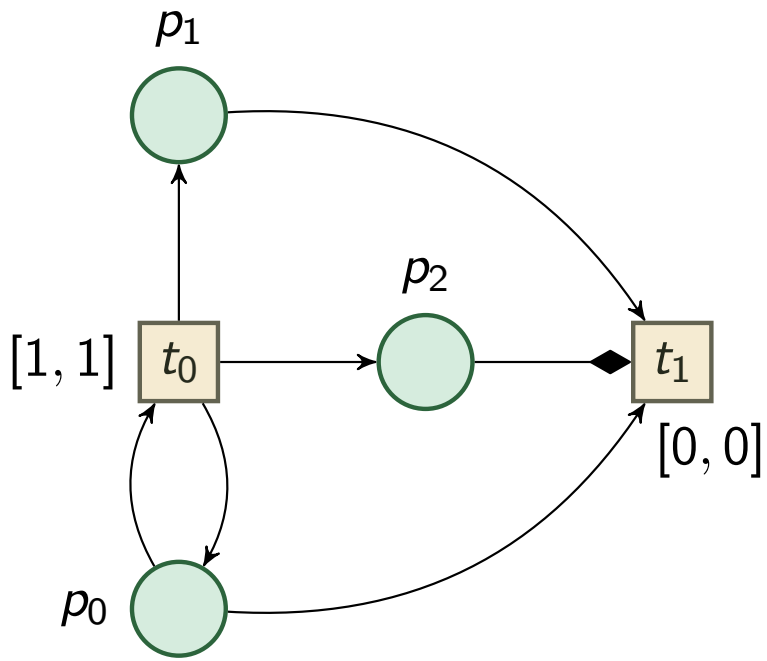
$$\begin{array}{lll} t_0 \in [1,1] & \xrightarrow{1} & [0,0] \\ t_1 \in - & & - \end{array}$$

Reset and transfer arcs



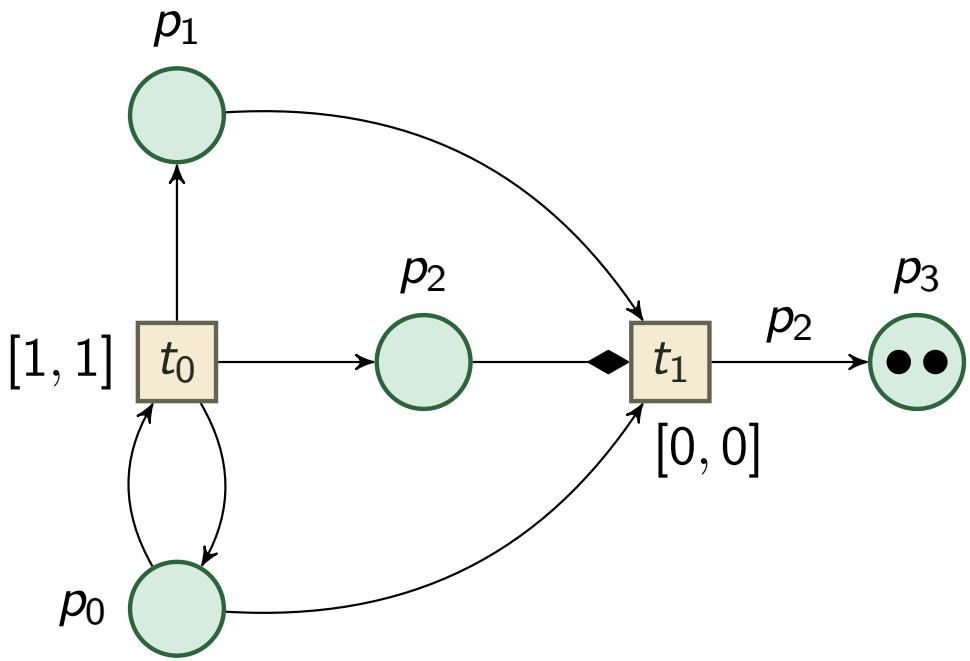
$$\begin{array}{lcccc} t_0 \in [1,1] & \xrightarrow{1} & [0,0] & \xrightarrow{t_0} & [1,1] \\ t_1 \in - & & - & & [0,0] \end{array}$$

Reset and transfer arcs



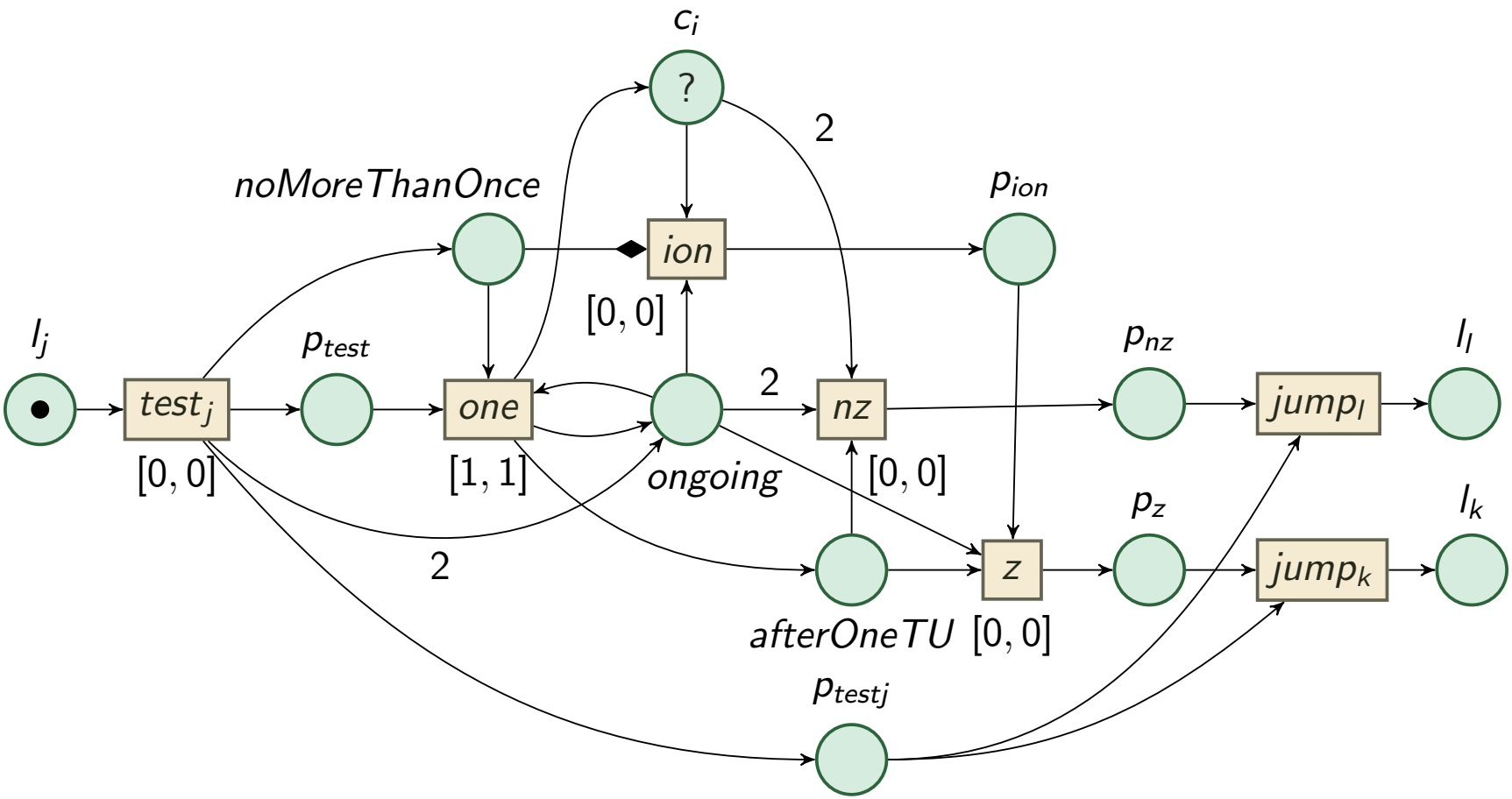
$$\begin{array}{l} t_0 \in [1,1] \\ t_1 \in - \end{array} \xrightarrow{1} \begin{array}{l} [0,0] \\ - \end{array} \xrightarrow{t_0} \begin{array}{l} [1,1] \\ [0,0] \end{array} \xrightarrow{t_1} \begin{array}{l} - \\ - \end{array}$$

Reset and **transfer** arcs

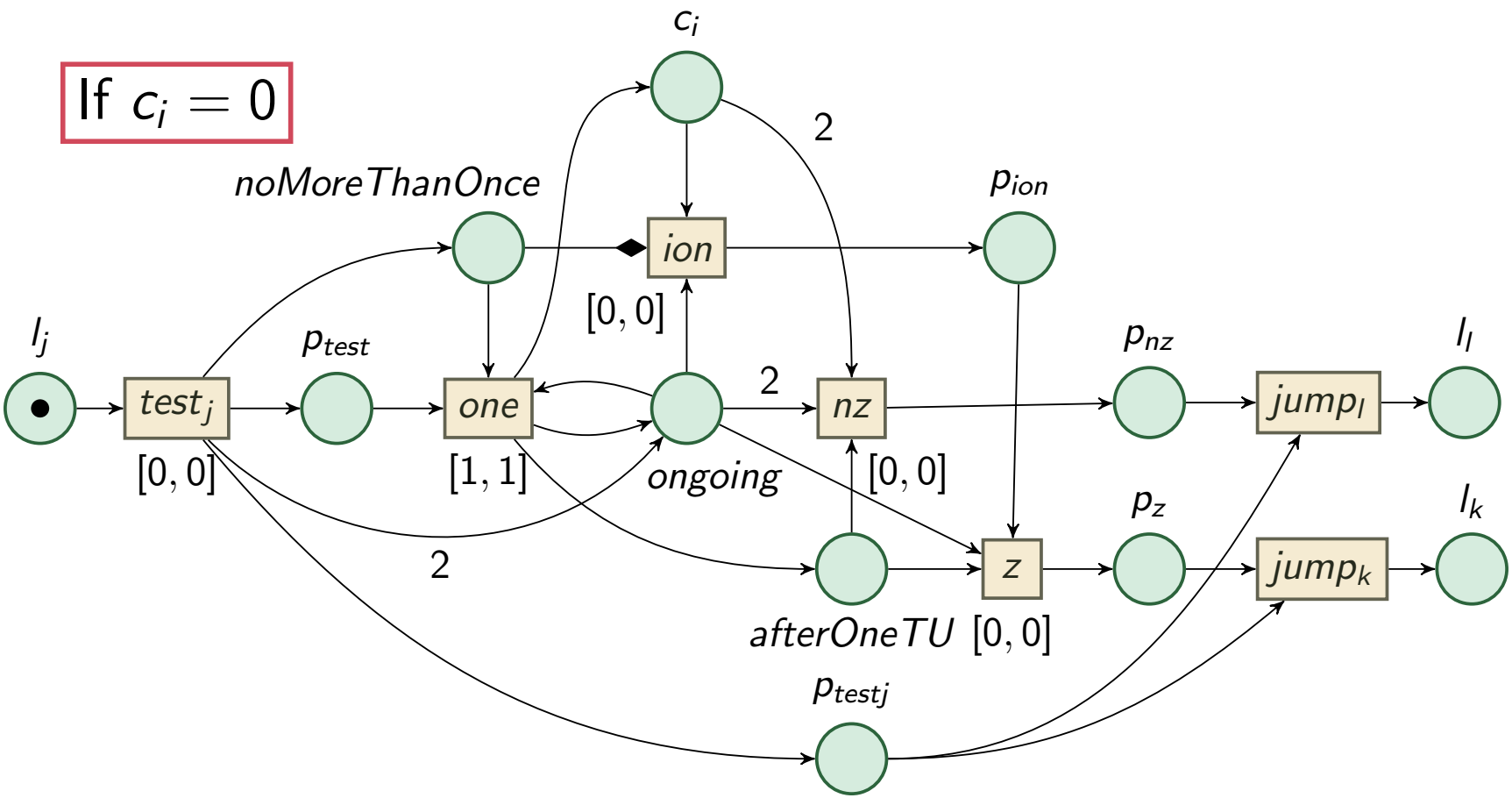


$$\begin{array}{l} t_0 \in [1,1] \\ t_1 \in - \end{array} \xrightarrow{1} \begin{array}{l} [0,0] \\ - \end{array} \xrightarrow{t_0} \begin{array}{l} [1,1] \\ [0,0] \end{array} \xrightarrow{t_1} \begin{array}{l} - \\ - \end{array}$$

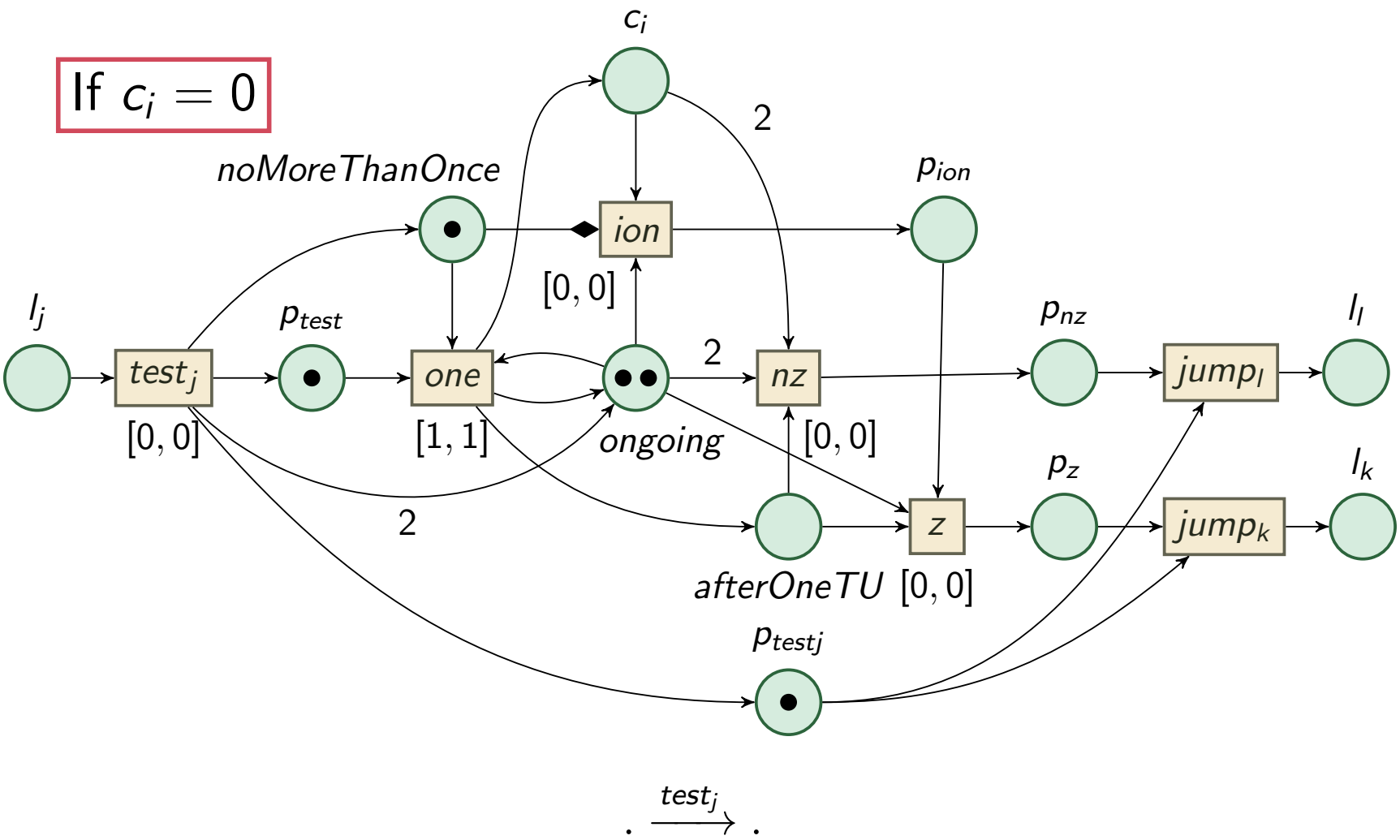
Zero-test with reset arc



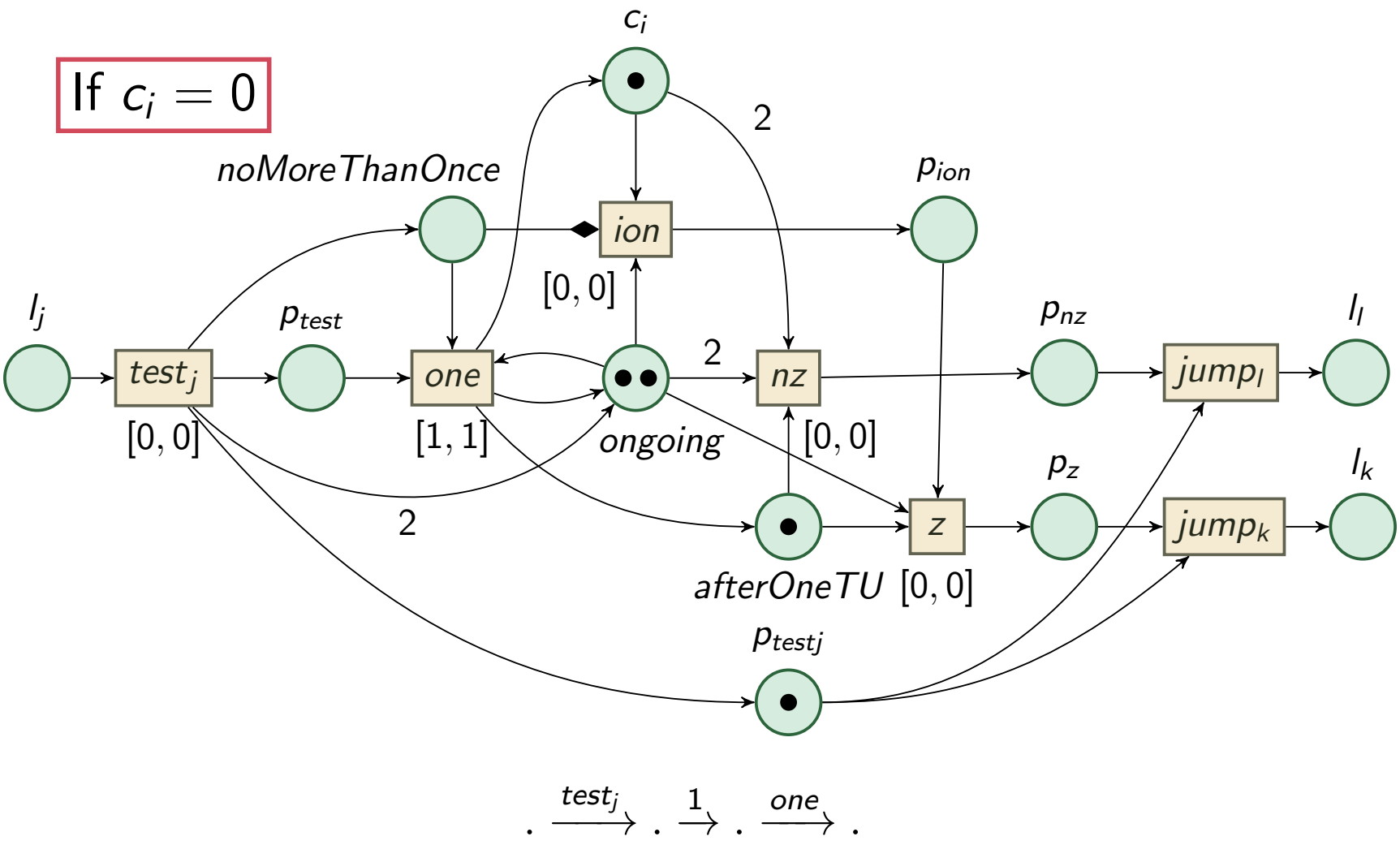
Zero-test with reset arc



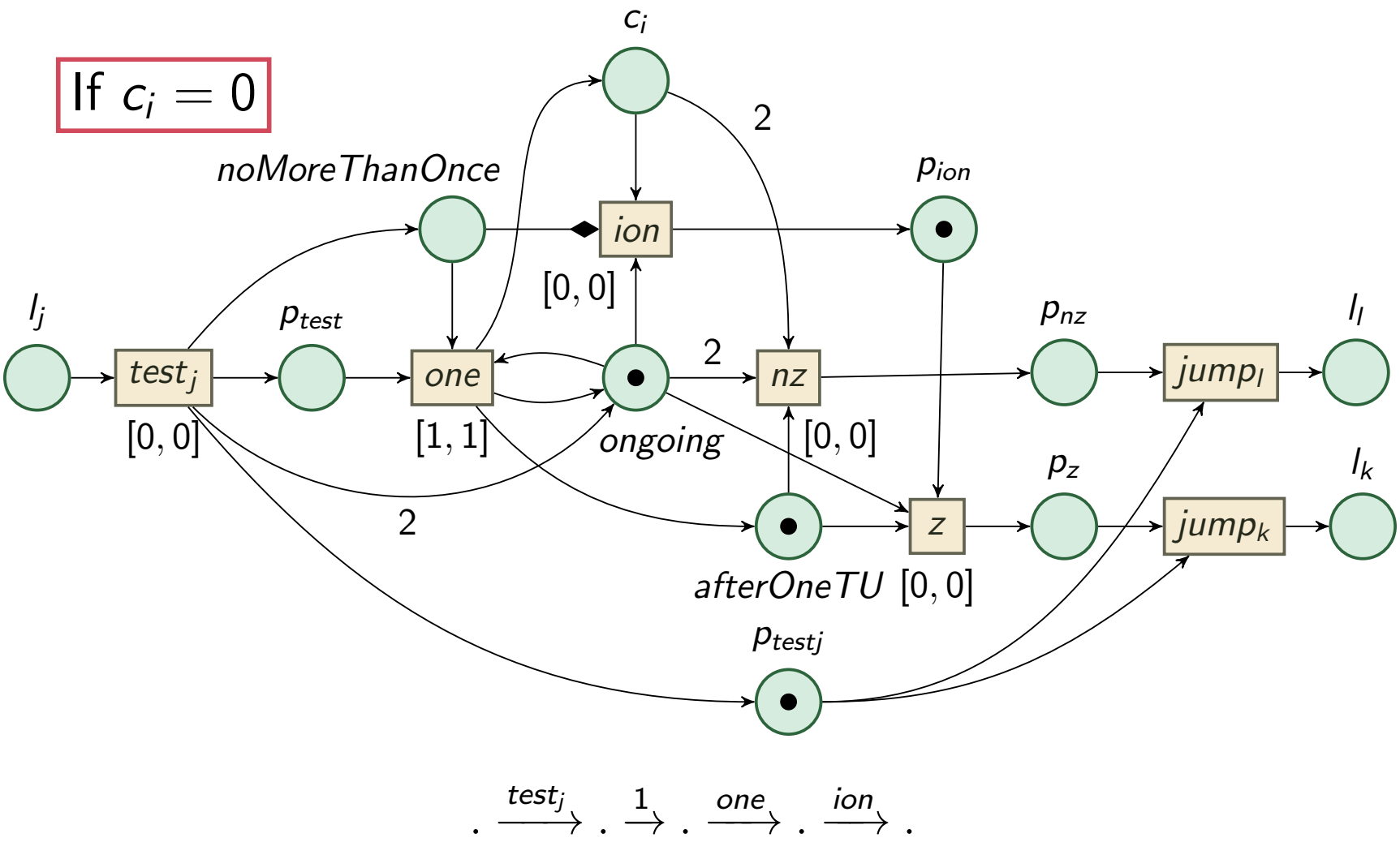
Zero-test with reset arc



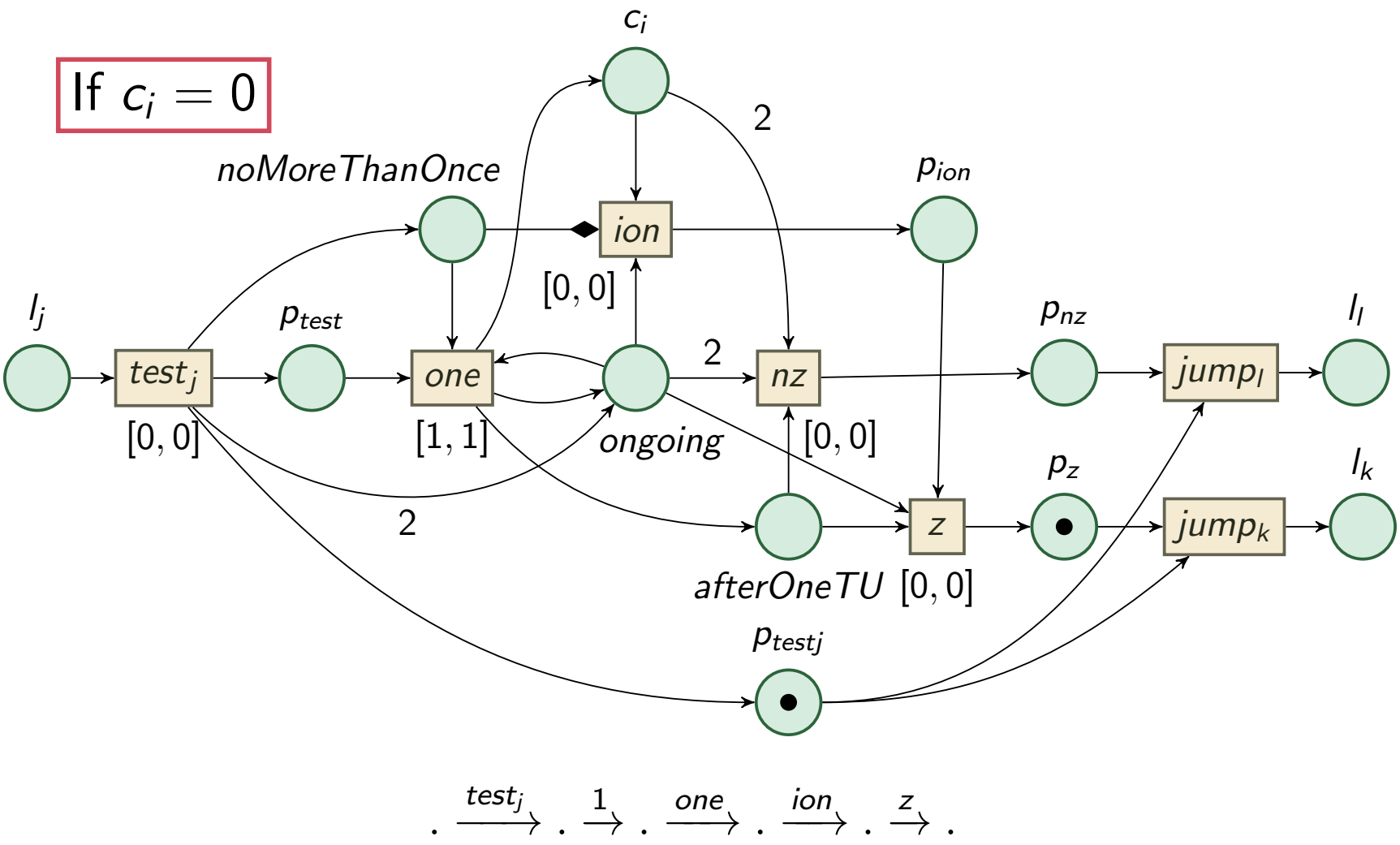
Zero-test with reset arc



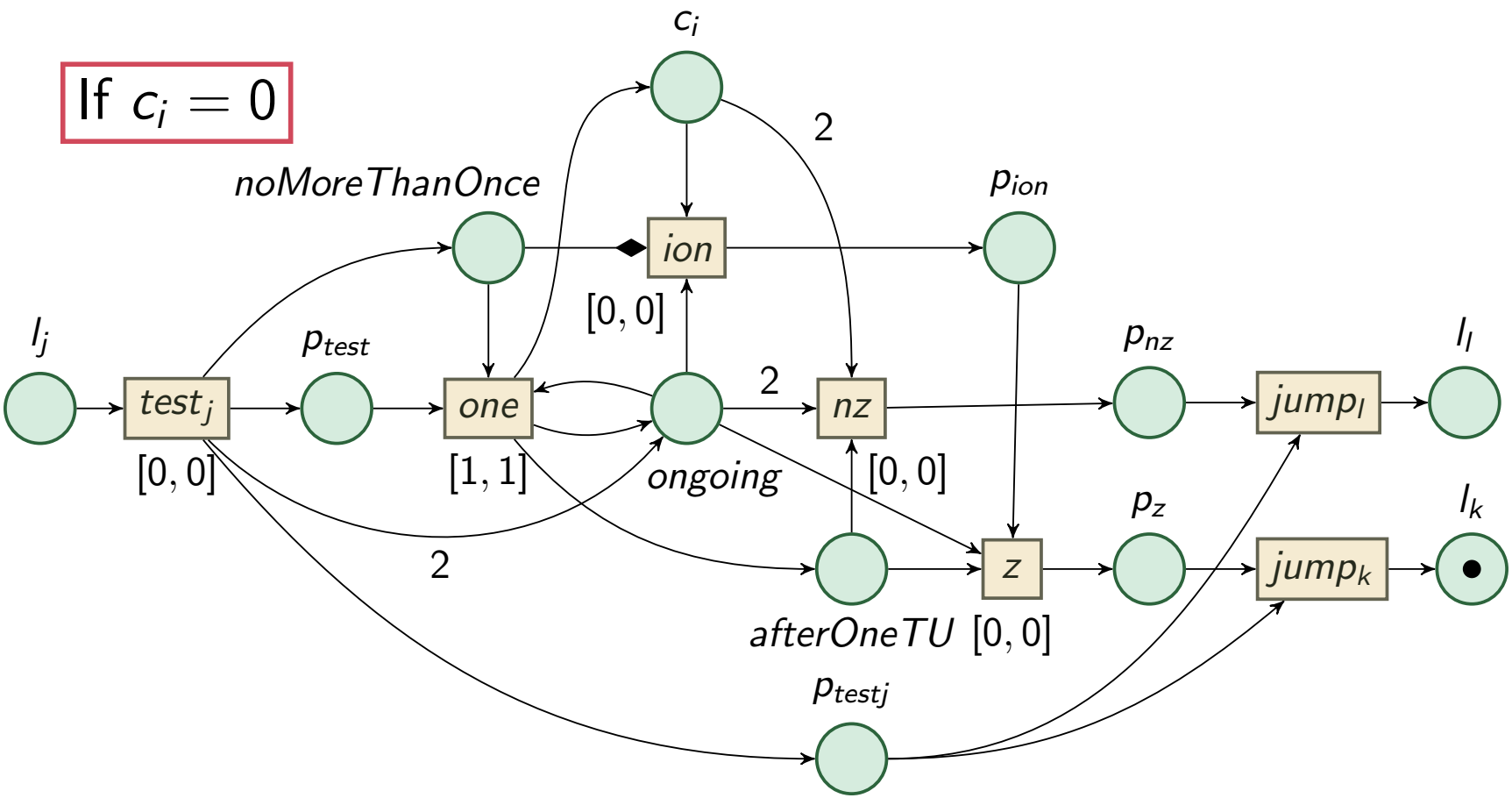
Zero-test with reset arc



Zero-test with reset arc

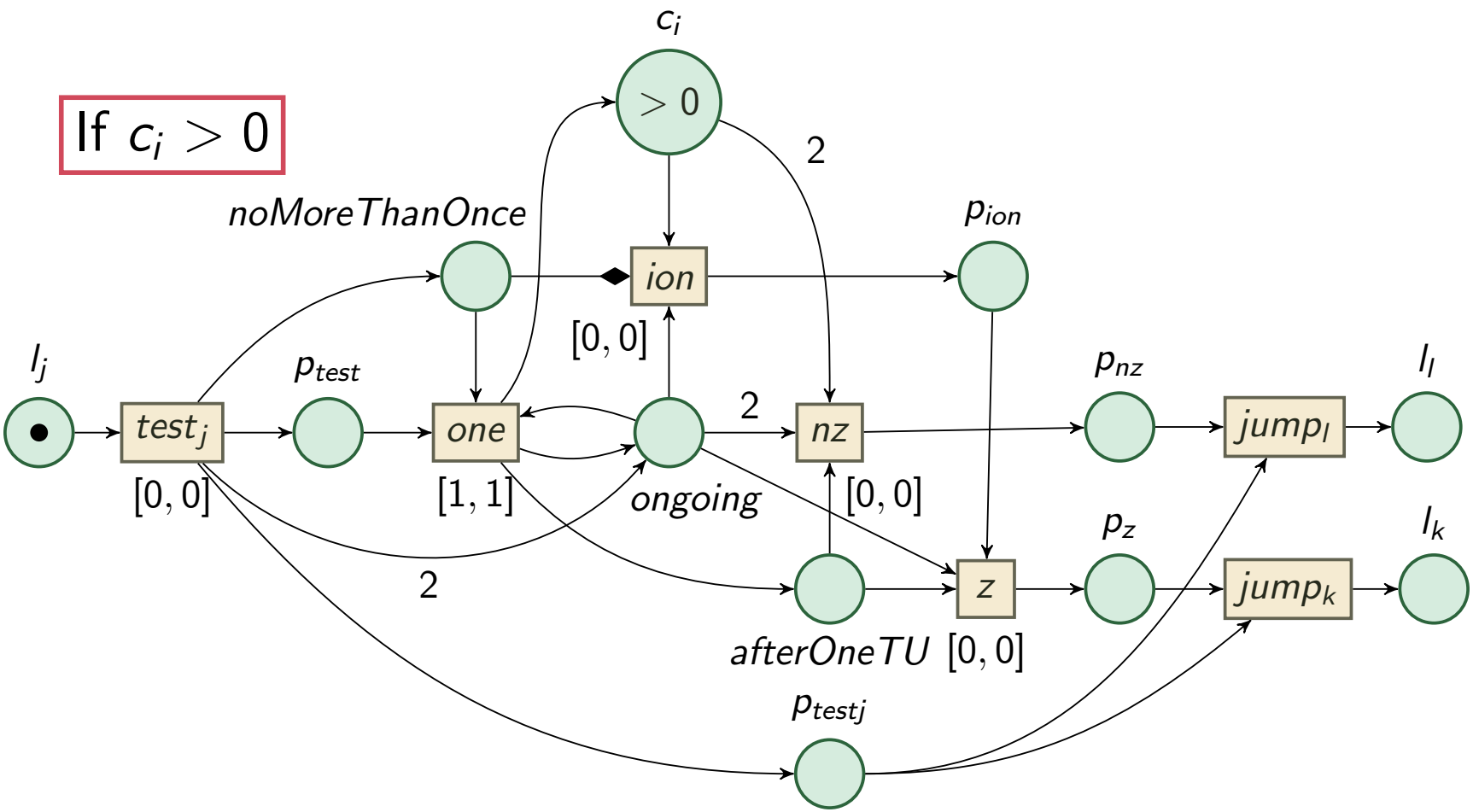


Zero-test with reset arc

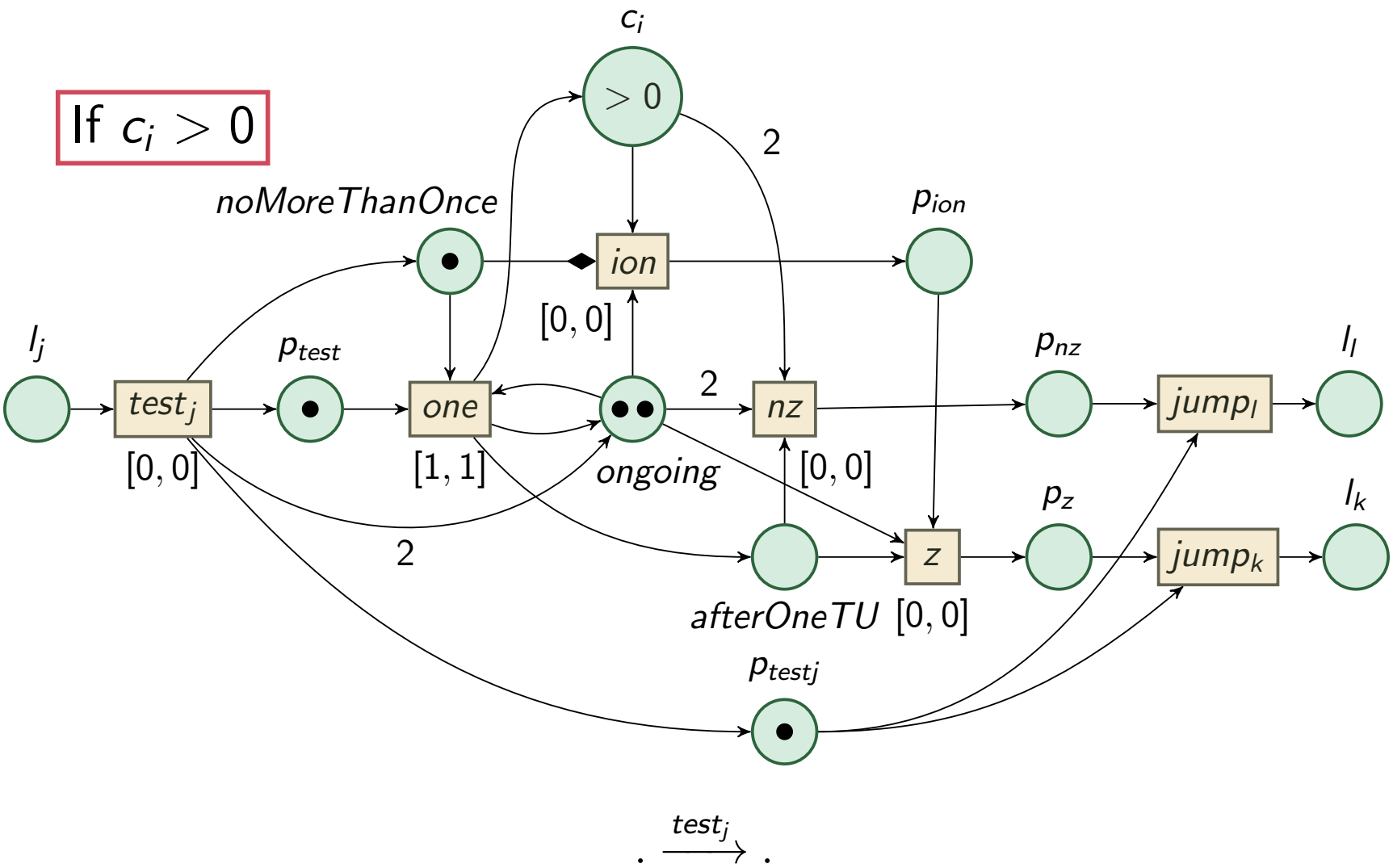


$$\cdot \xrightarrow{test_j} \cdot \xrightarrow{1} \cdot \xrightarrow{one} \cdot \xrightarrow{ion} \cdot \xrightarrow{z} \cdot \xrightarrow{jump_k} \cdot$$

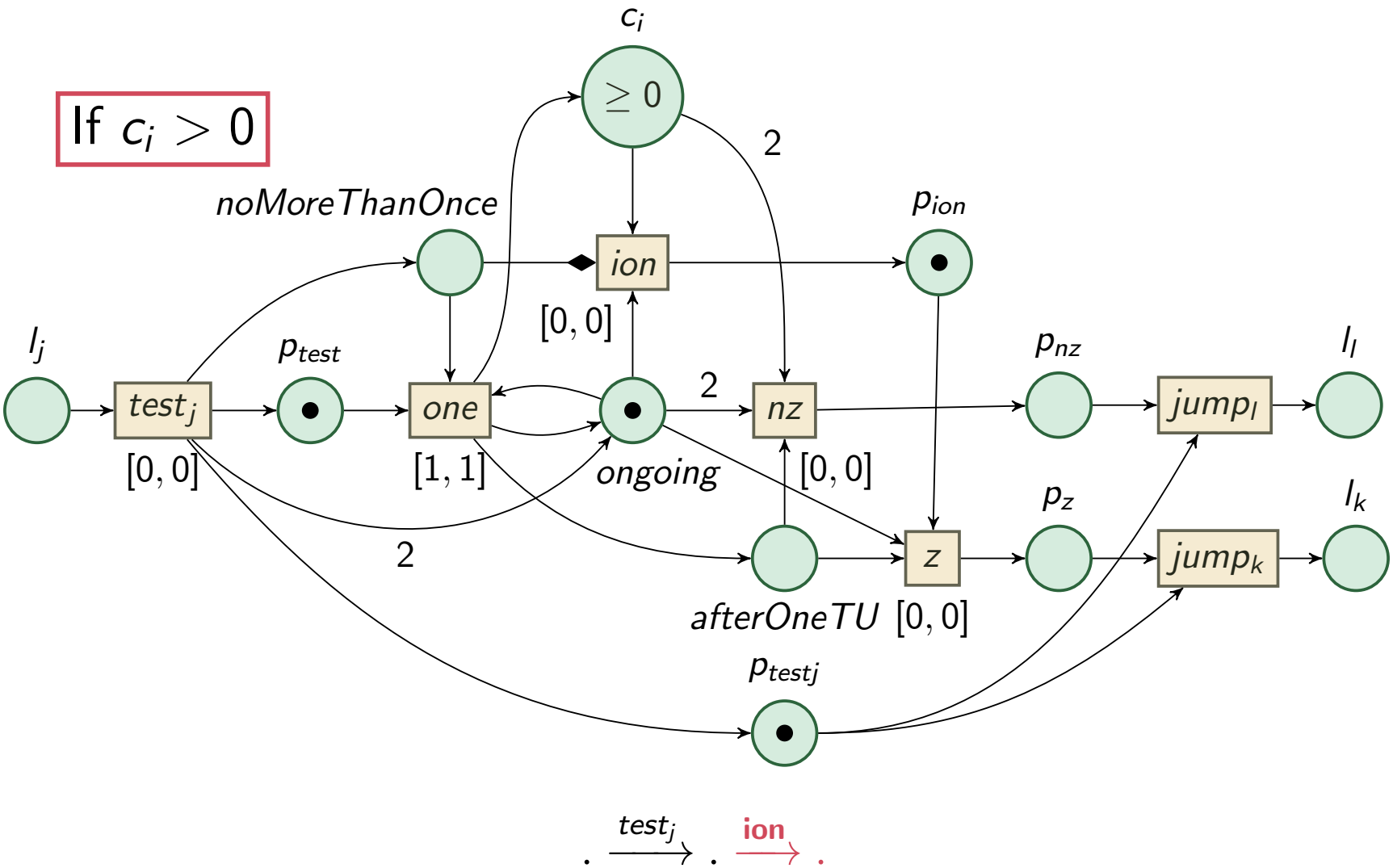
Zero-test with reset arc



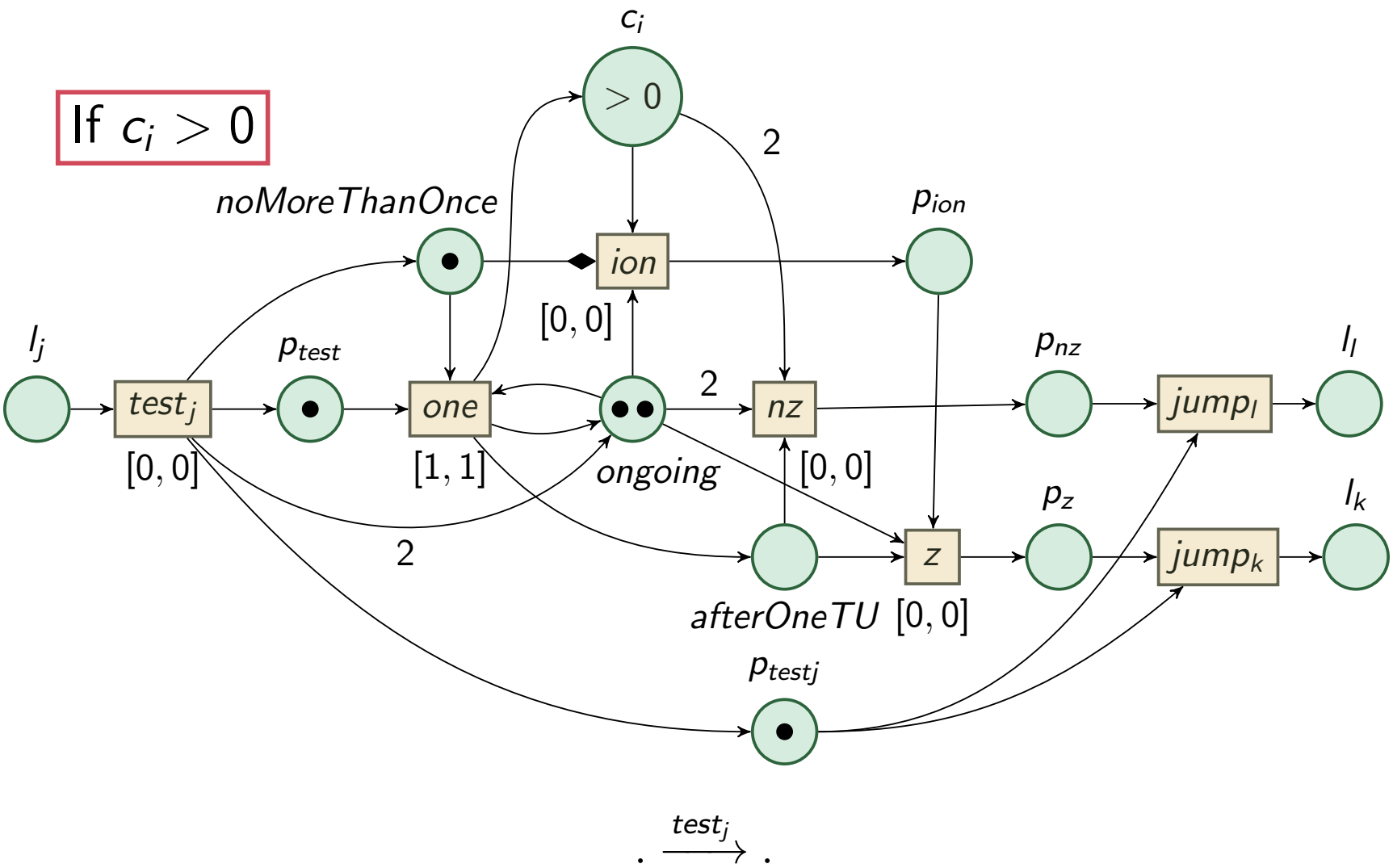
Zero-test with reset arc



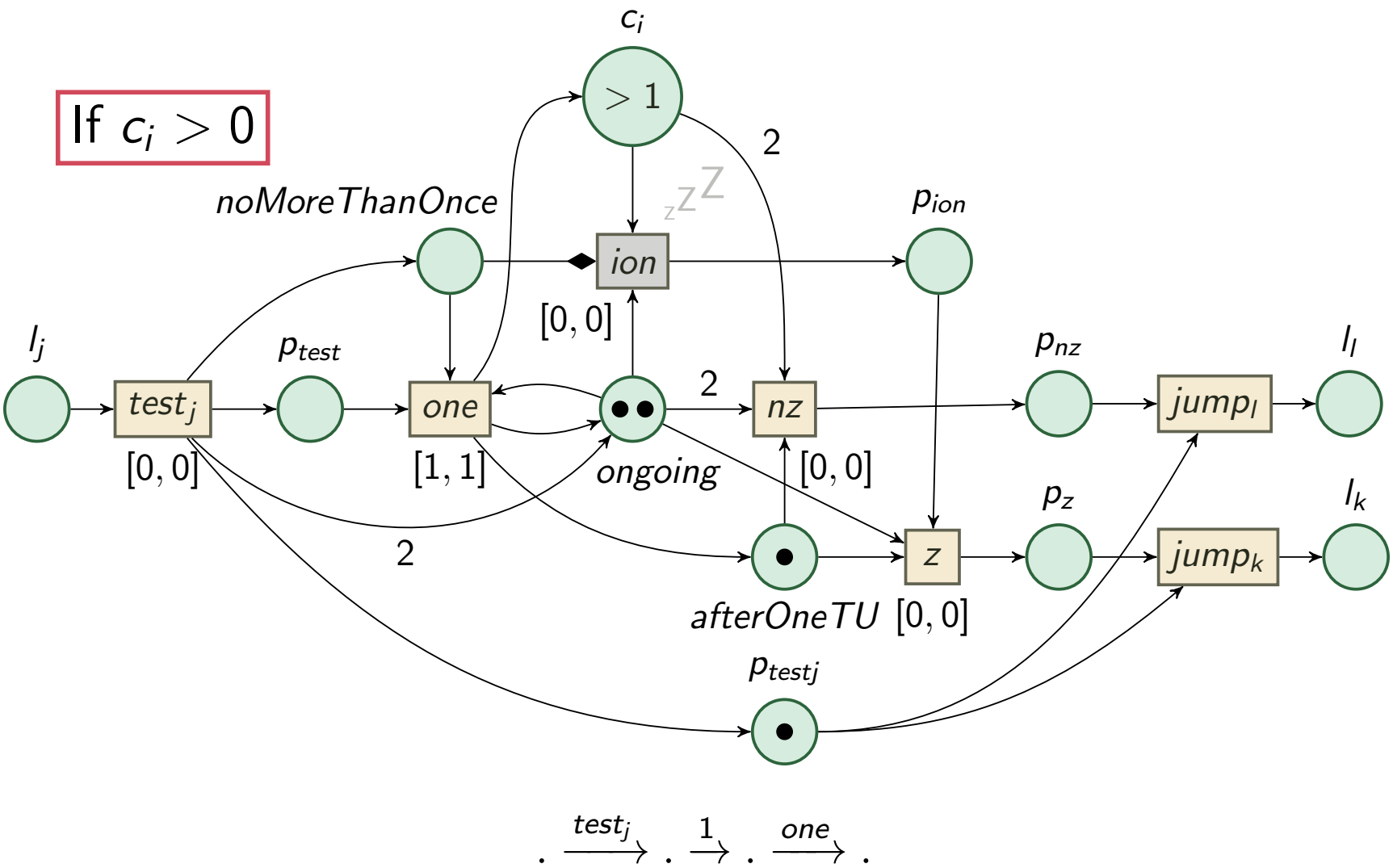
Zero-test with reset arc



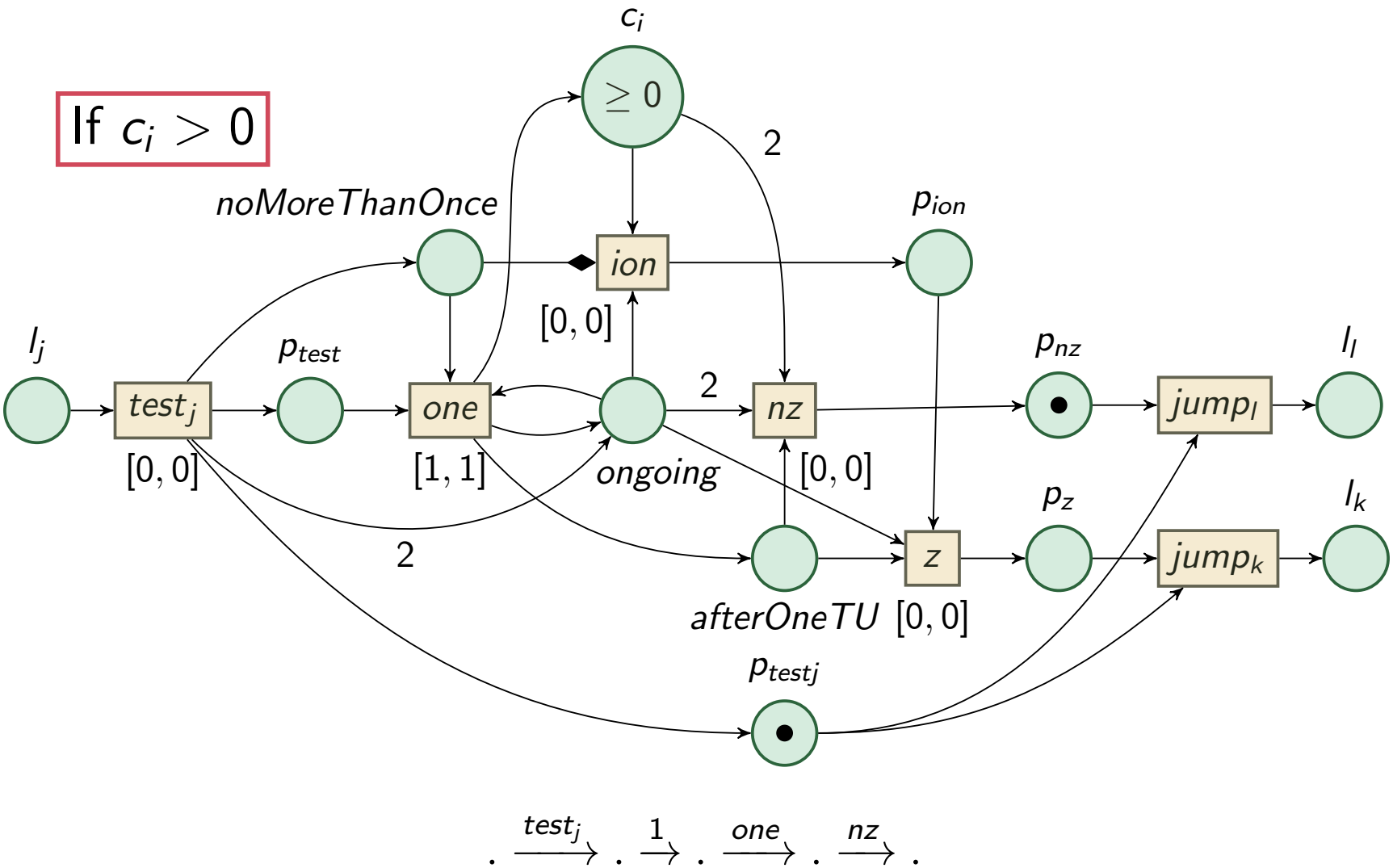
Zero-test with reset arc



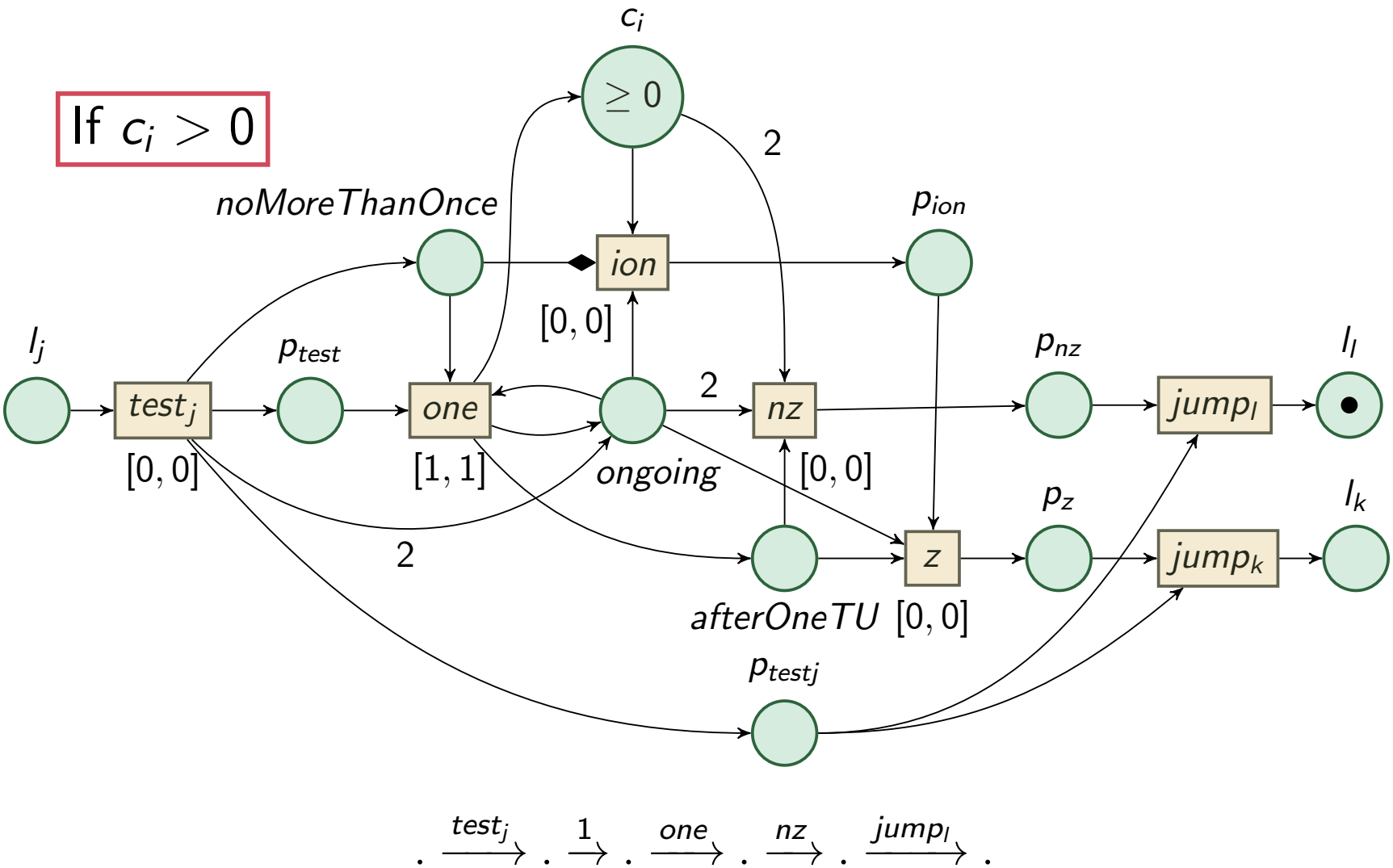
Zero-test with reset arc



Zero-test with reset arc



Zero-test with reset arc



Outline

Weak and strong semantics for T-TPN

Weakly-strong TPN

Weak TPN with read, reset or transfer arcs

Conclusion

Conclusion and Future Work

Summary

- Reachability, coverability and boundedness are *undecidable* for
  - weak TPN + 1 strong transition
  - weak TPN + 2 read arcs
  - weak TPN + 2 reset arcs
  - weak TPN + 2 transfer arcs
- Definition of State Class Graph for bounded WS-TPN
  - Polyhedral representation of *firing time* and *sleeping time*

Future work

- Refine decidability results: e.g. without conflicts on strong transitions;
- Refine expressiveness results: e.g. timed languages;
- Implement in Roméo.